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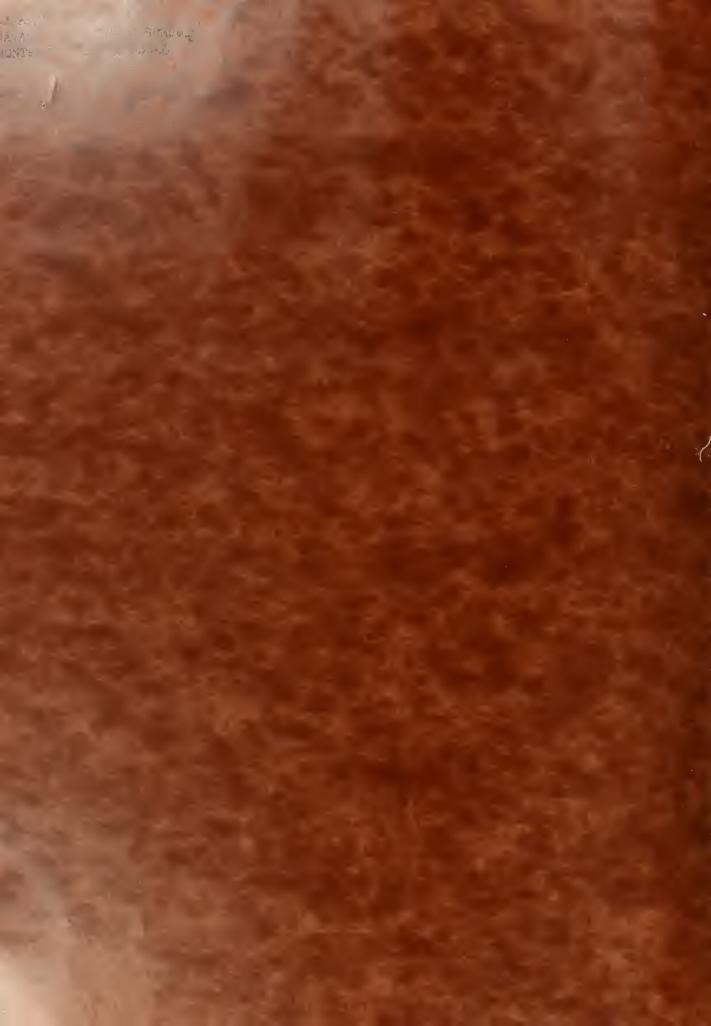
EMERGENCY ASCENT TRAJECTORIES

FOR DEEP SUBMERSIBLES

by

HERBERT WILLIAM TUFTS, III

June, 1969



FOR DEEP SUBMERSIBLES

by

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B.S., Virginia Polytechnic Institute

(1963)

SUBSTITTED IN PARTIAL FULFILLENT

OF THE REQUIREMENTS FOR THE DEGREE OF

NAVAL ENGINEER

AND THE DEGREE OF

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at the

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ABSTRACT

EMERGENCY ASCENT TRAJECTORIES
FOR DEEP SUBMERSIBLES

by

HERBERT WILLIAM TUFTS, III

Submitted to the Department of Naval Architecture and Marine Engineering on May 21, 1969 in partial fulfillment of the requirements for the degree of Naval Engineer and the degree of Master of Science in Ocean Engineering.

After a brief discussion of the need for predicting the emergency ascent trajectory of a submersible and the means by which a mathematical model of an ascent can be derived, the second order, coupled equations of motion for a vehicle with varying mass and center of mass are derived.

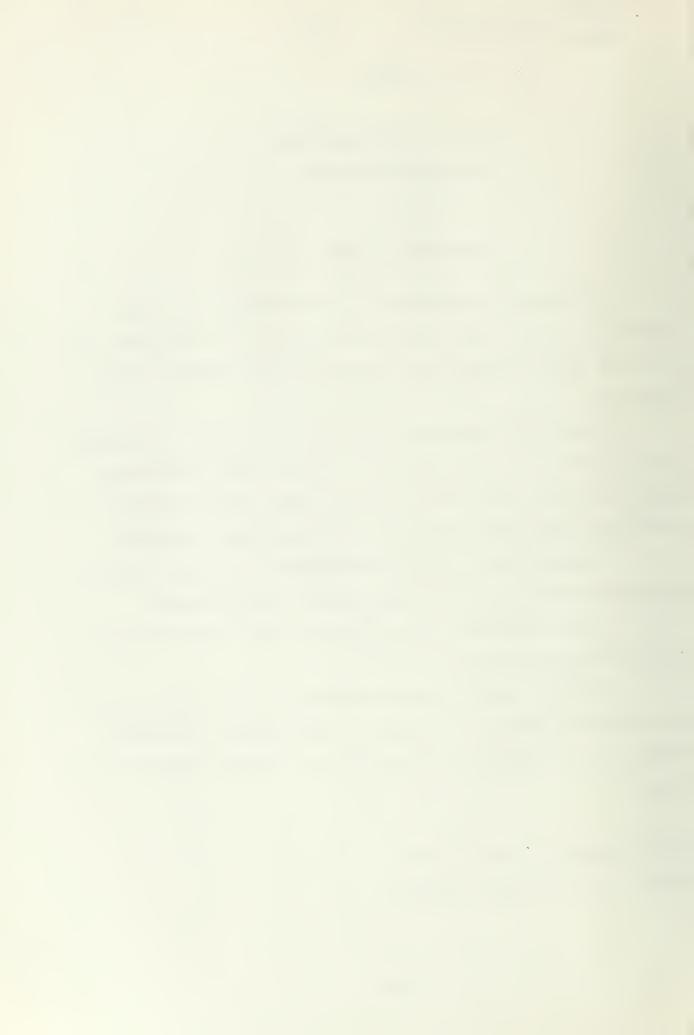
The equations of motion are then solved by a numerical step-wise procedure which is ameanable to programming on a digital computer.

Ascent trajectories are calculated using data from model experiments on the Deep Submergence Rescue Vehicle.

The tests indicate that two dimensional, vertical plane, equations are all that are necessary to determine an ascent through an undisturbed medium, but are insufficient once the vehicle is disturbed from its vertical plane.

Thesis Supervisor: Martin A. Abkowitz

Title: Professor of Naval Architecture



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I wish to admowledge the advice and assistance of the supervisor of this thesis, Professor Martin A. Abkowitz. I would particularly like to express my thanks for the opportunity to work under his direction.

The assistance and encouragement of Commander Sherman Reed in my dealings with NSRDC are greatfully acknowledged.

All computer programs were written for and executed by the IBM 360 Digital Computer of the Information Processing Center of Massachusetts Institute of Technology.



TABLE OF CONTENTS

TITLE PAGE	j.
ABSTRACT	ii
ACKNOVLEDGELENTS	iii
TABLE OF CONTENTS	iv
NOMENCIATURE	vi
INTRODUCTION	1
CHAPTER I: FOREULATION OF THE PROBLEM	5
I - 1 Assumptions	5
I - 2 Initial and Other Conditions	6
CHAPTER II: EQUATIONS OF MOTION	7
II - 1 General	7
II - 2 Axis Systems	7
II - 3 Conservation of Momentum Equations	11
II - 4 Gravity Forces	20
II - 5 Hydrodynamic Forces	21
II - 6 Equations of Motion for Free Ascent	- 41
II - 7 Summary	45
CHAPTER III: SOLUTION OF THE EQUATIONS OF MOTION	46
III - 1 Revised Equations of Motion	47
III - 2 Sterwise Linear Solution	52
III - 3 One Dimensional Ascent Trajectory	54
III - 4 Sumary	57
CHAPTER IV: RESULTS AND CONCLUSIONS	58
IV = 1 Results of Commuter Simulations	5.0



IV - 2 Conclusions and Recommendations	62
REFERENCES	63
APPENDIX A: COMPUTER PROGRAM FOR ASCENT TRAJECTORIES	65
A - 1 General.	65
A - 2 Input - Output	68
A - 3 Description of Namelists	72
A - 4 Program Listings	78
APPENDIX B: TEST VEHICLE GEOMETRIC AND CONTROL CHARACTERISTICS	94
APPENDIX C: INPUT AND OUTPUT FOR TEST VEHICLE	101
APPENDTY D: A PROGRAM TO COMPUTE ONE DISTRIBUTIONAL ASCENT TRANSCEPTES	105

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NOMENCLATURE

Symbol	Dimensionless Form	Definition
AB, AE		Body and earth fixed axis
		systems respectively
В	$B^{\bullet} = B/\frac{1}{2}\rho 1^2 U^2$	Buoyancy force, positive up-
		ward
СВ		Center of buoyancy of submers-
		ible
CG		Center of gravity or mass of
		submersible
I _G , I _o		Inertial tensor (3 x 3) about
	·	vohicle CG and vehicle ori-
		gin respectively
Ixx	$I'_{xx} = I_{xx}/\frac{1}{2}\rho 1^5$	Moment of inertia of submers-
		ible about x axis
I	$I'_{yy} = I_{yy}^{\frac{1}{2}\rho}1^{\frac{5}{2}}$	Moment of inertia of submers-
		ible about y axis
Izz	$\mathbf{I'}_{\mathbf{z}\mathbf{z}} = \mathbf{I}_{\mathbf{z}\mathbf{z}} / \pm \rho 1^5$	Noment of inertia of submers-
		ible about z axis
ı	$I'_{xy} = I_{xy}/_{2}\rho 1^{5}$	Product of inertia of submers-
		ible about xy plane



1 nz $I'_{xz} = I_{xz} / \frac{1}{2} \rho 1^5$

Product of inertia of submersible about xz plano

 $\mathbf{I}_{\mathbf{y}z}$

 $T_{yz} = I_{yz} / \frac{1}{2} \rho I^5$

Product of inertia of submersible about yz plane

K, H, N

 $K' = K/\frac{1}{2}\rho 1^3 U^2$

Rolling, pitching and yawing moments respectively

K

 $K'_{u} = K_{u} / \frac{1}{2} \rho 1^{3} U = \frac{3}{2} K' / \frac{3}{2} u'$

Typical static moment derivative; derivative of a moment component with respect to a velocity component,

K

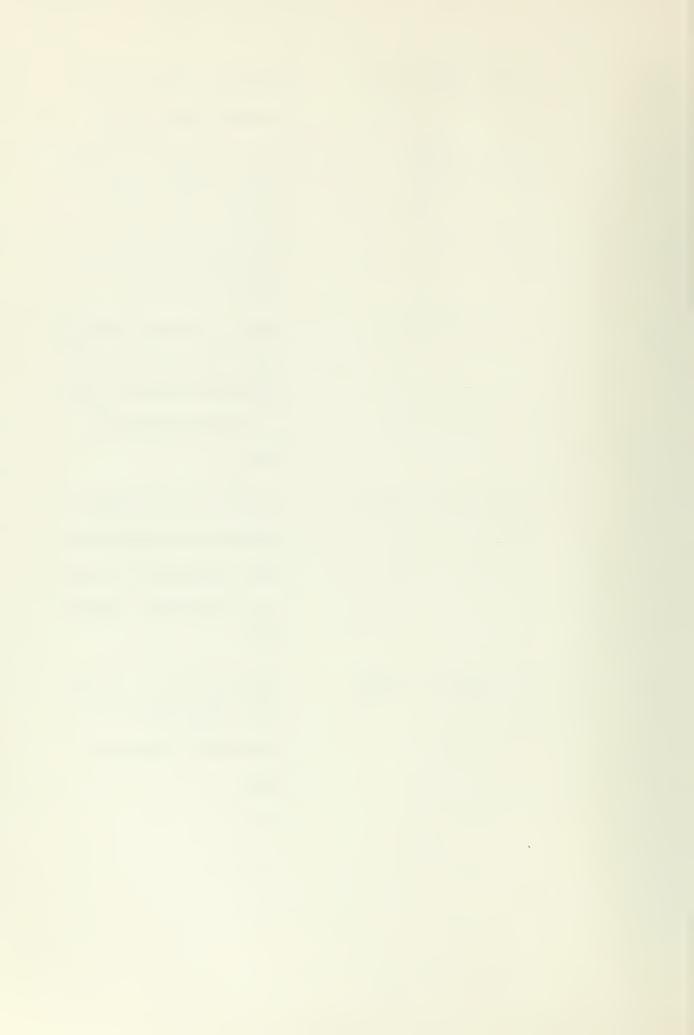
 $K'_{\dot{\mathbf{u}}} = K_{\dot{\mathbf{u}}} / \frac{1}{2} \rho \mathbf{1}^{l_{\dot{\mathbf{v}}}} = \delta K' / \delta \dot{\mathbf{u}}'$

Typical moment of inertia coefficient; derivative of a moment component with respect to an acceleration component, aK/au

Кр

 $K'_{p} = K_{p} / \frac{1}{2} r l^{l_{p}} U = \partial K' / \partial p'$

Typical rotary mon t derivative with respect to an angular velocity component, 3K/3p



К.

 $K_{\bullet}^{b} = K^{\bullet} \sqrt{2}bI_{2} = 9K_{\bullet} / 9b_{\bullet}$

Typical moment of inertia coefficient; derivative of a
moment component with respect
to an angular acceleration
component, 2 K/2 p

K

 $K_{\Lambda} = K^{\Lambda \Lambda} / \frac{1}{2} b I_3 = 2 K_1 / 2 K_1 / 3 K_1$

Typical second order moment coefficient; derivative of a moment component with respect to the square of a velocity component or the product of two velocity components, $2^{2}K/_{2}V_{3}W$

K va $K'_{vq} = K_{vq}/\frac{1}{2}\rho I^{4} = S^{K'}/\delta v' \delta q'$

Typical second order moment coefficient; derivative of a moment component with respect to the product of a velocity component with an angular velocity component, $\frac{2}{5}K/\frac{2}{5}V_{3}Q$

K

 $K'_{pq} = K_{pq}/\frac{1}{2}\rho I^{5} = \frac{2}{3}K'/\delta p \delta q$

Typical second order moment coefficient; derivative of a moment component with respect to the square of an angular velocity component or the product of two angular velocity components, $\frac{2}{5}K/_{5}p_{3}q$



K.,	$K_{*} = K_{*}/2\rho 1^{3}u^{2}$	Typical moment coefficient
		when body angles (a, \beta) and
		control surface angles are
		zero
3.	- l' = 1/l = 1	Characteristic length of the
		submersible
	3	
m -	$m^{\bullet} = m/\frac{3}{2}\rho 1^{3}$	Mass of submersible, includ-
		ing water in ballast tanks
O		Origin of body axes
p, q, r	p' = pl/U	Angular velocities of roll,
	· -	pitch and yaw, respectively
p, q, r	$\dot{\mathbf{p}}' = \dot{\mathbf{p}} 1^2 / \mathbf{u}^2$	Angular accelerations of roll,
		pitch, and yaw, respectively
t	t' = t U/1	Timo
TA, TB		Angular and translational
A D		velocity transformation
		matrices, respectively; trans-
		formation is from body to
	•	earth axes
m m		Vin his and a contract
Tf, Tv		Kinetic energy of fluid and

vohicle, respectively



 $u' = u/\overline{U}$ Longitudinal, transverse and u, v, w normal components, respectively of the velocity of the origin of body axes relative to the fluid 11 = 11/02 ů, v, v Longitudinal, transverse and normal components, respectively of the acceleration of the origin of body axes relative to the fluid U' = U/U = 1Velocity of origin of body axes relative to the fluid -- V' = V/13 Volume of submersible $W' = W/\frac{1}{2} 1^2 U^2$ Weight of submersible W x' = x/lx, y, z Body axes, or coordinates of a point relative to body axes $x_B, y_B, z_B \qquad x'_B = x_B/1$ Coordinates of the center of buoyancy relative to body axes

Coordinates of the center of

gravity relative to body axes

 $x_G, y_G, z_G \qquad x_G = x_G/1$



$$x_0$$
, y_0 , z_0 $x'_0 = x_0/\lambda$

$$\dot{x}_{E}$$
, \dot{y}_{E} , \dot{z}_{E} \dot{x}_{E} = \dot{x}_{E}/U

$$X, Y, Z \qquad X' = X/\frac{1}{2} \rho 1^2 U^2$$

$$X_{u} \qquad X_{\dot{u}} = X_{u} / \frac{1}{2} \rho 1^{2} U = \delta X^{\prime} / \delta u^{\prime}$$

$$X' = X_{\bullet} / \frac{1}{2} \ell 1^{3} = \partial X' / \partial \mathring{u}'$$

X.

$$X_i^b = X^b \sqrt{5}b J_3 \cap = 9X_i / 5b_i$$

Fixed or inertial axes, or coordinates of a point relative to fixed axes

Longitudinal, transverse and normal components, respective—
ly of the velocity of the origin of body axes relative to the inertial axes

Longitudinal, lateral and normal components, respectively, of hydrodynamic force on the submersible

Typical static force derivative of a force component with respect to a velocity component, $\frac{3}{2}$

Typical inertia coefficient; derivative of a force component with respect to an acceleration component, $2X/2\hat{u}$

Typical rotary force derivative; derivative of a force component with respect to an angular velocity component,



Х. р

$$X^{b} = X^{b} / \frac{1}{2} / \frac{1}{2} / \frac{1}{4} = 9X^{c} / 9b^{c}$$

Typical inertia coefficient;
derivative of a force component with respect to an angular acceleration component, $\partial X/\partial \hat{p}$

X uw

$$X'_{uv} = X_{uv}^{1/2} \rho I^{2} = \frac{2}{8} X' / \delta u \delta w$$

Typical second order force coefficient; derivative of a force component with respect to the square of a velocity component or the product of two velocity components,

X

$$X^{\bullet}_{Vr}/\frac{1}{2}\rho 1^3 = 2^2X^{\bullet}/\epsilon_{V} r$$

Typical second order force coefficient; derivative of a force component with respect to the product of a velocity component, $\frac{\partial^2 X}{\partial v \partial r}$

X

$$\mathbf{X'_{rp}} = \mathbf{X_{rp}}/\mathbf{z_{fl}} = \mathbf{z_{x'/arap}}$$

Typical second order force coefficient; derivative of a force component with respect to the square of an angular velocity component or the product of two angular velocity components, 2X/2r2p



 $X'_{EFF} = X_{EFF} / \frac{1}{2} \rho 1^2 u^2$ X_{EFF} $Y'_* = Y'_* / \frac{1}{2} \rho L^2 U^2$ Yx , Zx d.B 0, 4, 6 Vector Time derivative 3/3t Non-dimensionalized parameter [11] Matrix

Typical effector force coefficient

Lateral and normal forces
when body angles (%,6) and
control surface angles are
zero

Angles of attack and drift,
respectively

Specific weight of the fluid
Angles of pitch, yaw and roll
respectively

Mass density of the fluid

Derivative

Partial derivative

6

d



INTRODUCTION

An important operating mode of deep submersibles is the ascending and descending mode. Unlike large high speed military submarines which ascend and descend dynamically by use of a combination of speed and appendage deflection, deep submersibles depend partially and sometimes entirely, as in the case of the TRIESTE, upon buoyant ascent and descent. Of the ascending and descending operations, the most critical is during an emergency ascent in which most or all of the jettisonable weights are removed from the vehicle in order to obtain as rapid an ascent as possible.

The majority of deep submersibles fall into a class of vehicles characterized by low speed and minimal appendage streamlining. They are designed for underwater research and rescue missions which do not, except for the travel mode of the Deep Submergence Rescue Vehicle, require large speeds. They are equipped with many and diverse appendages for accomplishing their missions such as manipulator arms, lights, propulsion motors, sampling equipment, TV cameras and mating skirts, all of which effect, to varying degrees, the streamlining and hydrodynamics of these vehicles.

In order to study the motions and ascent trajectories of submersibles in normal or emergency ascent without model or full scale tests,
it is necessary to develop a mathematical model of the vehicle which can
accommodate the effects of varying mass and center of mass.

This work is an attempt to develop the mathematical model using a set of second order equations of motion, and solve these equations by use of a high speed digital computer.

The primary sources of material for this work are the works of Abkowitz, Dogan, Gertler and Hagen, Lamb and Strumpf, but the secondary



sources, acknowledged or not, are no less important of the development of this work.

The problem of determining the motions of a totally submerged body has been treated by many authors, however, there appear to be only two basic means for arriving at these equations. These are by energy methods and by vector calculus.

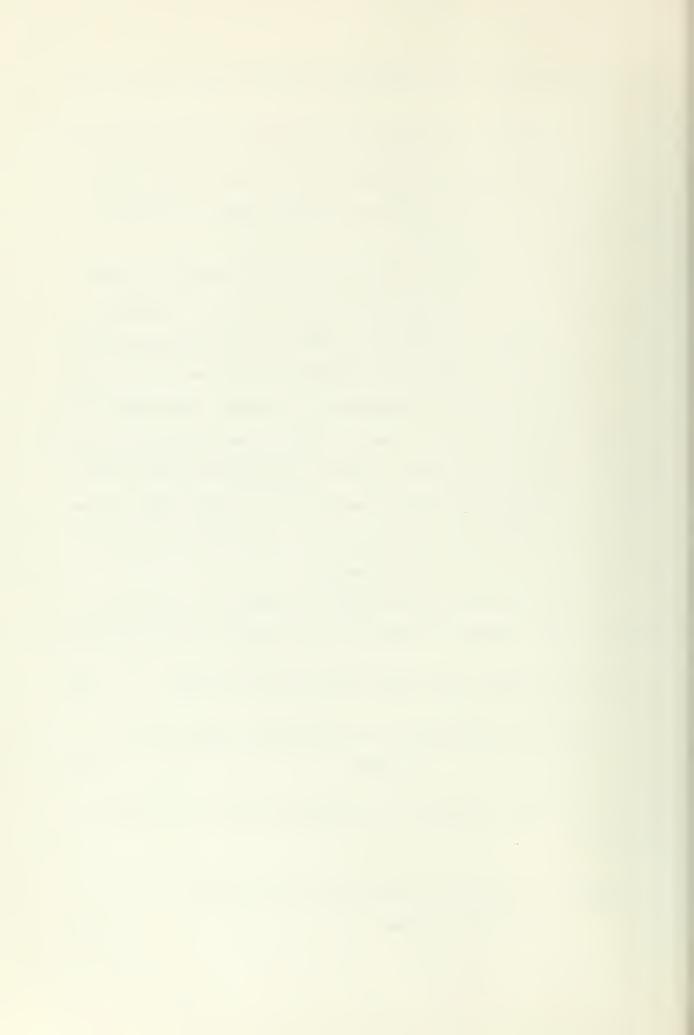
The first method is that used by Sir Horace Lamb in his book "Hydrodynamics" which first appeared in 1879 (see ref 1). The energy method is based upon the existance of a single valued velocity-potential which implies that the motion of the fluid is entirely due to that of the submersible, and is therefore irrotational and acyclic. This method leads to a completely general set of equations representing the rigid body dynamics, but the hydrodynamic forces that result represent only the so called added mass or added inertia effects and not all the forces acting on the body. Of the forces that are missing the most important are those due to circulation, separation and vortex shedding.

The second method of developing the equations of motion is to combine a vector expansion of Newton's laws of motion, expressed as follows:

$$\overline{F} = \frac{d}{dt}$$
 (Momentum) = the vector force acting on the body (1)

$$\vec{h} = \frac{d}{dt}$$
 (Angular momentum) = the vector moment acting on the body (2)

with a Taylor series expansion of the hydrodynamic forces and moments expressed as:



which reduces to

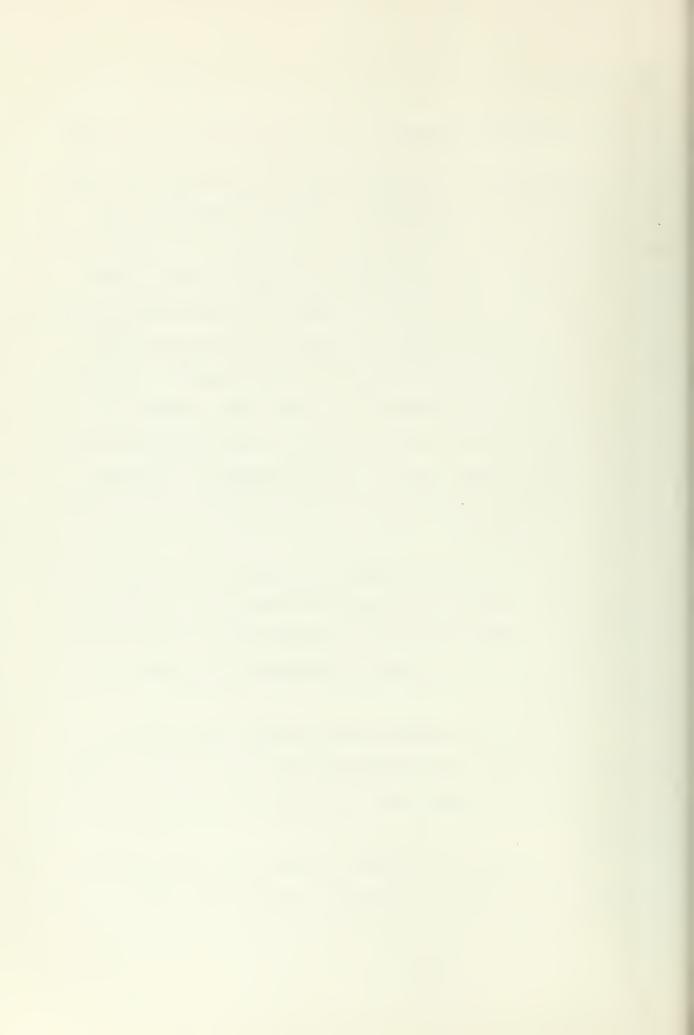
$$\frac{\vec{F}}{M} = \hat{f}$$
 (Properties of notion) (4)

for a particular vehicle in a particular fluid. This method has been used by A. Strumpf in 1960 in developing his "Equations of Motion of a Submerged Body with Varying Mass" (see ref 2) and by Prof. M. A. Abkowitz in 1949 in his lecture course and notes on "The Dynamical Stability of Submarines" (see ref's 3 and 10). It too results in a completely general set of equations describing the rigid body dynamics, but its greatest asset lies in the generality of the hydrodynamic forces and moments. The hydrodynamic forces and moments obtained from a Taylor series expansion include not only all the added mass effects but also the circulation, separation and vortex shedding effects. The only limitation on this method is the availability of theoretical or experimental data to use in the equations.

From the preceeding discussion and equations (1 through 4), it is obvious that the derivation of the rigid body motion may be completely separated from the problem of developing a suitable form for the hydrodynamic effects. Therefore, the two methods of development may be broken down and part of each used.

This work shall attempt to take advantage of the simplicity of the energy development of the rigid body dynamics, while retaining the generality provided by the Taylor series expansion of the hydrodynamic forces and moments.

The solution of the equations of motion that will be developed employs a technique suggested in 1964 by G. Parissis in his solution of linearized equations of motion for heave and pitch of a ship (see ref 4)



and again in 1967 by L. M. McCloskey for the digital simulation of the DSRV control system and autopilot (see ref 5).

The coefficients used to test the solution of the equations of motion come from a series of model experiments conducted on the DSRV. The results of the computer solution will not, however, be the ascent trajectory for the DSRV since there is an assumption, in this work, of zero propulsion forces in the development of the equations of motion.

The assumptions and the statement of the problem are given in Chapter I, the equations of motion are derived in Chapter II and a method for solving the equations of motion is developed in Chapter III.



CHAPTER I

FORMULATION OF THE PROBLEM

The problem involves a deep submersible vehicle of given geometry and dimensions which is in free ascent through a stationary fluid.

I - 1 Assumptions

The vehicle is assumed to be a rigid body with no clastic deformations of a vibratory nature. It has six degrees of freedom, three translational and three rotational. We shall be interested in motions in both the horizontal and vertical planes of motion. Velocities are small. Hydrodynamic effects of second order in acceleration shall be considered negligible. The only body symmetry is port and starboard.

I - 1.1

The neglecting of vibratory motions is reasonable since the frequencies of vibration of the hull acting as an elastic body are of different orders of magnitude than the frequencies of motion and do not excite the latter. Elastic deformations due to vehicle compressability must be included since they directly affect the buoyancy of the vehicle.

I - 1.2

The interest in both planes of motion is due to the desire for as general a set of equations as possible.

I - 1.3

The assumption of small velocities is realistic in that free ascent velocities and open ocean currents are in general of order less than ten knots.



I - 1.4

The use of a set of second order equations of notion is deemed necessary to appropriately describe the hydrodynamic cross-coupling that takes place in a problem such as this. The second order acceleration effects are assumed to be zero on the basis of potential theory (see ref 1). This, however, has not been experimentally verified.

I - 1.5

Neglecting assymmotries due to relatively small appendages which are not control surfaces, there are few vehicles which operate in the ocean environment which do not possess port and starboard symmetry. For this reason the assumption of port and starboard symmetry has been made.

I - 2 Initial and Other Conditions

The vehicle is to be initially at rest relative to the inertial, earth fixed, axes. The control effectors, propellers, thrusters, rudders, dive planes, etc., are inoperable and/or in neutral position. The driving force for the vehicle shall be a decrease in weight due to jettisoning of ballast or an increase in volume due to vehicle decompression as it rises.

These conditions, though arbitrary, serve to restrict the problem to one of manageable size.



CHAPTER II

EQUATIONS OF MOTION

II - 1 Goneral:

The derivation of the equations of motion for a rigid body in six degrees of freedom with a varying mass and center of mass follows closely that of Dr. Pierre Dogan in reference (5), which is based primarily upon the development of the equations of motion by Lamb in reference (1). The derivation of the hydrodynamic force equations follows that of Professor Martin Abkowitz in reference (3) except that, in this work, the second order terms are retained. The forces due to gravity are determined using the transformations set forth in reference (6) and the angular velocity transformations given by Professor Abkowitz in reference (7).

The axis systems necessary to describe the motions of a body and its trajectory through a fluid include a body fixed system and an inertial system.

II - 2 Axis Systems

The equations of motion for an ascending vehicle must be written in an earth fixed axis system in order to determine the trajectory of the vehicle relative to some fixed point. An additional body fixed system is required in order to describe the hydrodynamic interactions between the vehicle and the water.

II - 2.1 Inertial Axis System

The right-handed earth fixed axis system, $\Lambda_{\rm E}$ (see figure II.1), is assumed to be an inertial system for the reason that the accelerations



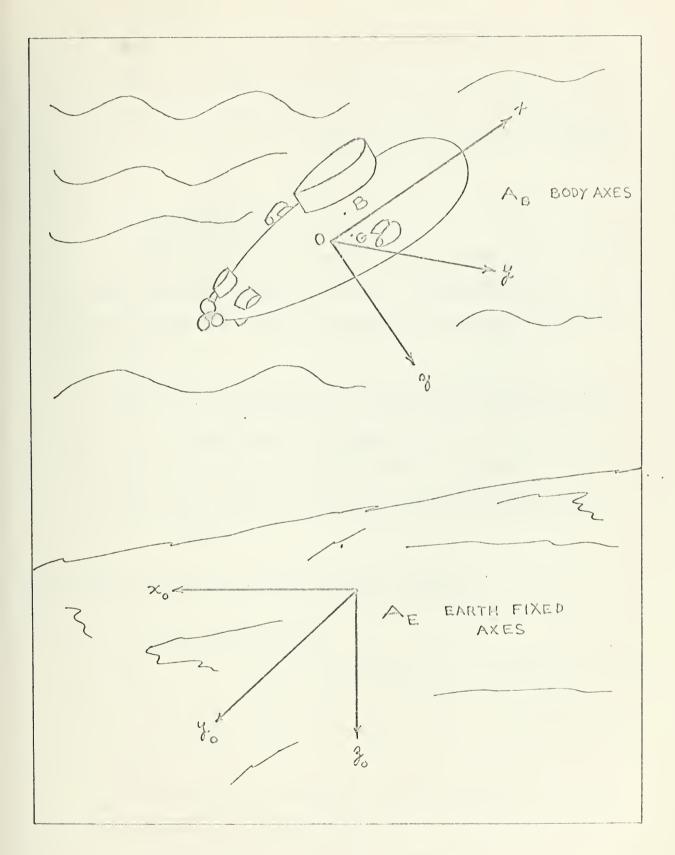


Figure 1 Coordinate Systems



of a point on the surface of the earth are an order of magnitude smaller than those which are of importance to the motions of the vehicle. $A_{\rm E}$ is an orthogonal set of axes fixed relative to the earth such that components $x_{\rm E}$ and $y_{\rm E}$ are in a horizontal plane, and the $z_{\rm E}$ axis is vertical and directed downwards.

II - 2.2 Body Axis System

The right-handed body axis system, $A_{\rm B}$ (see figure II.1), is fixed in the vehicle such that advantage is taken of the assumed principal plane of symmetry by placing the origin of the system in this plane. The axes of this system are:

- x axis the longitudinal axis, directed from the after to the forward end of the vehicle,
- y axis the transverse axis, directed to starboard,
- z axis the normal axis, directed from top to bottom (deck to keel).

Tho xz plane is the assumed principal plane of symmetry.

II - 2.3 Body Axes Orientation

Angular displacements of $A_{\rm B}$ relative to $A_{\rm E}$ are specified by a set of modified 'Euler angles' which are taken as positive in the sense of rotation of a right-handed screw advancing in the positive direction of the axis of rotation.

The orientation of $\Lambda_{\rm B}$ relative to $\Lambda_{\rm E}$ is described in terms of a roll angle ϕ , a pitch angle θ and a yaw angle ψ . Before defining these angles, an order of rotations must be chosen since finite rotations are not true vector quantities and do not obey the rules for adding vectors. The



order chosen here conforms to that set forth in reference (6) which is:

- (1) rotate about the initial $z=z_{\rm E}$ exis through an angle of yaw γ .
- (2) rotate about the new position of the $y = y_1$ axis through an angle of pitch 0,
- (3) finally rotate about the new position of the x = x axis through an angle of roll \(\ell \).

In accordance with the order of rotations above we have the following definitions:

- θ the angle of pitch; the angle of elevation of the x axis; the angle between the x axis and the horizontal plane $x_{E}y_{E}$.
- u the angle of yaw; the angle from the vertical plane $\mathbf{x}_{\mathrm{E}}\mathbf{z}_{\mathrm{E}}$ to the vertical plane \mathbf{x}_{E} ;
- ϕ the angle of roll; the angle from the vertical xz_E plane to the principal plane of symmetry xz.

The successive rotations required to specify the orientation of the body axes relative to the earth fixed axes can be described by three orthogonal matrices $\left[\psi\right]_{Z_{E}}$, $\left[0\right]_{y_{1}}$, $\left[\phi\right]_{x}$:

$$\begin{bmatrix} \psi \end{bmatrix}_{Z_{E}} = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos 0 & 0 & -\sin 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos 0 & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin 0 & 0 & \cos \theta \end{bmatrix}$$



$$\begin{bmatrix} \phi \end{bmatrix}_{\mathbf{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

Unit vectors in Λ_{R} and Λ_{R} are related by an orthogonal matrix:

$$\begin{bmatrix} \mathbf{\dot{i}}_{\mathrm{B}} \\ \mathbf{\dot{j}}_{\mathrm{B}} \\ \mathbf{k}_{\mathrm{B}} \end{bmatrix} = \mathbf{T}_{\mathrm{B}} \begin{bmatrix} \mathbf{\dot{i}}_{\mathrm{E}} \\ \mathbf{\dot{j}}_{\mathrm{E}} \\ \mathbf{k}_{\mathrm{E}} \end{bmatrix}$$

where T_B is the product of the three orthogonal matrices defining the rotations:

$$\mathbf{T}_{\mathrm{B}} = \left[\phi\right]_{\mathrm{x}}\left[\theta\right]_{\mathrm{y}_{1}}\left[\psi\right]_{\mathrm{z}_{\mathrm{E}}} \quad .$$

$$= \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ -\cos \phi \sin \psi & \cos \phi \cos \psi & \sin \phi \cos \theta \\ +\sin \phi \sin \theta \cos \psi & +\sin \phi \sin \theta \sin \psi & \cos \phi \cos \theta \\ +\sin \theta \cos \phi \cos \psi & +\cos \phi \sin \theta \sin \psi & \end{bmatrix}$$

Velocities in ${\bf A}_{\rm B}$ and ${\bf A}_{\rm E}$ are also related by the transformation matrix ${\bf T}_{\rm B}$:

$$\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{v} \end{bmatrix} = \mathbf{T}_{\mathbf{B}} \begin{bmatrix} \mathbf{\dot{x}}_{\mathbf{E}} \\ \mathbf{\dot{y}}_{\mathbf{E}} \\ \mathbf{\dot{z}}_{\mathbf{E}} \end{bmatrix}$$

The vehicle angular velocities in $A_{\rm B}$ and $A_{\rm E}$ are, however, related by a non-orthogonal matrix, which is the sum of three components along the $z_{\rm E}$, $y_{\rm 1}$ and x axes of magnitude ψ , $\dot{\theta}$ and $\dot{\phi}$.



$$\begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \phi \\ \mathbf{x} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{vmatrix} \phi \\ \mathbf{x} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{vmatrix} \phi \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \phi \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & -\sin \theta \\ \cos \phi & \cos \theta \sin \phi \\ \mathbf{0} & -\sin \phi & \cos \theta \cos \phi \end{bmatrix} \cdot \begin{bmatrix} \phi \\ \mathbf{0} \\ \mathbf{\dot{\gamma}} \end{bmatrix}$$

$$= \mathbf{T}_{\mathbf{A}} \begin{bmatrix} \phi \\ \mathbf{\dot{0}} \\ \mathbf{\dot{\gamma}} \end{bmatrix}$$

II - 3 Conservation of Momentum Equations

The derivation of the dynamical equations for a vehicle with a verying mass and center of mass have been treated in two different manners by:

- (1) Albert Strumpf of Davidson Lab, (see ref 2) using vector calculus, and including variations in mass, moments of inertia and CG position.
- (2) Pierre Dogan of MIT Instrumentation Lab, (see ref 5), using a Langrangian formalism, including time variation of the inertial tensor, and making assumptions as to the form of the movable weights.

The treatment by Strumpf assumes a body that is a rocket, so that mass is discharged from the body. The treatment by Dogan, for the Doep Submergence Rescue Vehicle (DSRV), assumes that the mass of the body is constant but allows the position of the center of gravity to change.

The development by Dogan avoids the long vectoral contrations



involved in Strumpf's development, while sacrificing generality by not including a change in mass. This feature could be included if it were desired and so the approach by Dogan will be used.

Nine generalized velocities and coordinates are sufficient to describe the motions of the vehicle. These are: six velocities (u, v, w, p, q, r) to describe the vehicle and three coordinates (x_G, y_G, z_G) to describe the motion of the center of gravity. The following six Lagrange equations will give the needed vehicle momentum equations. (See Lamb, "Hydrodynamics" 6th Ed, page 168)

$$\frac{d}{dt} \frac{\partial T_{v}}{\partial v} - r \frac{\partial T_{v}}{\partial v} + q \frac{\partial T_{v}}{\partial v} = X$$
 (2.3 - 1)

$$\frac{d}{dt}\frac{\partial T_{v}}{\partial v} - p\frac{\partial T_{v}}{\partial w} + r\frac{\partial T_{v}}{\partial u} = Y$$
 (2.3 - 2)

$$\frac{d}{dt} \frac{\partial T_{\mathbf{v}}}{\partial \mathbf{v}} - q \frac{\partial T_{\mathbf{v}}}{\partial \mathbf{v}} + p \frac{\partial T_{\mathbf{v}}}{\partial \mathbf{v}} = Z \tag{2.3 - 3}$$

$$\frac{d}{dt} \frac{\partial T_{v}}{\partial p} = v \frac{\partial T_{v}}{\partial v} + v \frac{\partial T_{v}}{\partial v} - r \frac{\partial T_{v}}{\partial v} + q \frac{\partial T_{v}}{\partial r} = K$$
 (2.3 - 4)

$$\frac{d}{dt} \frac{\partial T_{v}}{\partial q} - u \frac{\partial T_{v}}{\partial u} + v \frac{\partial T_{v}}{\partial u} - p \frac{\partial T_{v}}{\partial r} + r \frac{\partial T_{v}}{\partial p} = H$$
 (2.3 - 5)

$$\frac{d}{dt} \frac{\partial T_{v}}{\partial r} - v \frac{\partial T_{v}}{\partial u} + u \frac{\partial T_{v}}{\partial v} - q \frac{\partial T_{v}}{\partial p} + p \frac{\partial T_{v}}{\partial q} = N$$
 (2.3 - 6)

 T_{v} is the vehicle total kinetic energy. X, Y, Z, K, M, N are the generalized forces and moments of which some can be further described by a potential function and others represent friction and drag.

The three Lagrango equations necessary to describe the momentum balance equation for the jettisonable ballast subsystem are assumed to reduce to quasi-steady equations defining the CG location from the integrals of the various ballast release rates.

This proceedure is as much a necessity as it is a simplification



since, in the case of dropping ballast, the ballast may take almost any form from liquid mercury and iron shot to large blocks of metal or pieces of equipment. The dropping of a liquid or a granular solid can be reasonably modeled as a function of time but the dropping of chunks of metal or pieces of equipment would create singularities in a functional relationship. The obvious answer would be to use a combination of a smooth function and steps to obrain a reasonably accurate model of the deballasting of a vehicle.

There is one additional factor which also affects the decision to reduce the function to a quasi-steady process. The total ballast dropped is no more than three percent of the total vehicle weight and the rate at which it is removed is of order .3 percent of the total vehicle weight per second. This would then say that any Taylor series expansion of this function, which retained terms commesurate with the second order expansion to be used in obtaining the hydrodynamic forces, would contain at most the linear terms.

The effect of such an approximation is entirely dependent upon the length of the time interval over which the process is assumed to be steady and shall be discussed in conjunction with the computer program. Suffice it to say here, that until accurate model tests can be conducted, the effect of this assumption is truly unknown, but appears to be of inconsequential magnitude.

The equations describing the variable position of the CG in the body axis system are:



$$\mathbf{x}_{GB} V = \sum_{i}^{N} \mathbf{x}_{i} \sum_{i}^{t} V_{i} dt$$

$$\mathbf{x}_{G} = \sum_{i}^{N} V_{i}$$

$$W = \sum_{i}^{N} V_{i}$$

$$(2.3 - 7)$$

$$y_{GB} = \frac{V_{GB} - \sum_{i=1}^{N} y_{i} + \sum_{i=0}^{N} V_{i}}{V_{i}} = \frac{V_{GB} - \sum_{i=1}^{N} V_{i}}{V_{i}}$$
(2.3 - 8)

$$z_{G} = \frac{\sum_{i=1}^{N} z_{i} \sum_{i=1}^{t} w_{i}}{W - \sum_{i=1}^{N} w_{i}} dt$$

$$(2.3 - 9)$$

where \mathbf{x}_{GB} , \mathbf{y}_{GB} , \mathbf{z}_{GB} are the components of the CG of the vehicle with all of the ballast, W is the vehicle weight including all jettisonable ballast, $\mathbf{x}_{\mathbf{i}}$, $\mathbf{y}_{\mathbf{i}}$, $\mathbf{z}_{\mathbf{i}}$ are the CG's of the N jettisonable ballast weights $V_{\mathbf{i}}$. The vehicle kinetic energy is the sum:

$$T_{\mathbf{v}} = \frac{m}{2} \vec{V}_{G}^{2} + \frac{1}{2} \vec{\omega} I_{G} V \qquad (2.3 - 10)$$

where m, \vec{V}_G , I_G and $\vec{\omega}$ are the vehicle mass, the CG velocity, the inertial tensor about the CG and the angular velocity vector. Defining axes x', y', z' through the CG parallel to the vehicle body axes, one has

$$I_{G} = \begin{bmatrix} I_{x^{\dagger}x^{\dagger}} & I_{x^{\dagger}y^{\dagger}} & I_{x^{\dagger}z^{\dagger}} \\ I_{y^{\dagger}x^{\dagger}} & I_{y^{\dagger}y^{\dagger}} & I_{y^{\dagger}z^{\dagger}} \\ I_{z^{\dagger}x^{\dagger}} & I_{z^{\dagger}y^{\dagger}} & I_{z^{\dagger}z^{\dagger}} \end{bmatrix}$$
(2.3 - 11)

where I I y'y' have the usual meanings:



$$I_{x^*x^*} = \iiint_{V} (y^{*2} + z^{*2}) dn$$

$$I_{x^*y^*} = \iiint_{V} (x^*y^*) dn$$

Computing the components of \overline{V}_G in body axes one has:

$$\vec{V}_{G} = (u + qz_{G} - ry_{G} + \dot{x}_{G}, v + rx_{G} - pz_{G} + \dot{y}_{G}, u + py_{G} - qx_{G} + \dot{z}_{G})$$
(2.3 - 12)

Substituting (2.3 - 11) and (2.3 - 12) into (2.3 - 10) one gets the kinetic energy $T_{_{\mathbf{V}}}$ and its partial derivatives:

$$\frac{\partial T_{v}}{\partial u} = m \left(u + qz_{G} - ry_{G} + \dot{x}_{G} \right)$$

$$\frac{\partial T_{v}}{\partial v} = m \left(v + rz_{G} - pz_{G} + \dot{y}_{G} \right)$$

$$\frac{\partial T_{v}}{\partial v} = m \left(v + py_{G} - qz_{G} + \dot{z}_{G} \right)$$

$$\frac{\partial T_{v}}{\partial v} = m \left(v + py_{G} - qz_{G} + \dot{z}_{G} \right)$$

$$\frac{\partial T_{v}}{\partial v} = I_{x^{*}x^{*}} p + I_{x^{*}y^{*}} q + I_{x^{*}z^{*}} r$$

$$- nz_{G} \left(v + rx_{G} - pz_{G} + \dot{y}_{G} \right)$$

$$+ ry_{G} \left(v + py_{G} - qx_{G} + \dot{z}_{G} \right)$$

$$+ ry_{G} \left(v + py_{G} - qx_{G} + \dot{z}_{G} \right)$$

$$+ nz_{G} \left(u + py_{G} - qx_{G} + \dot{z}_{G} \right)$$

$$+ nz_{G} \left(u + qz_{G} - ry_{G} + \dot{x}_{G} \right)$$

$$\frac{\partial T_{v}}{\partial r} = I_{z^{*}z^{*}} r + I_{z^{*}x^{*}} p + I_{z^{*}y^{*}} q$$

$$- ry_{G} \left(u + qz_{G} - ry_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$

$$+ nz_{G} \left(v + rx_{G} - pz_{G} + \dot{x}_{G} \right)$$



The inertial tensor I_G about the CG can be algebraically related to the inertial tensor I_G about the origin (defined in x, y, z) and the distance x_G , y_G , z_G . For example:

$$I_{xx} = I_{x^*x^*} + m \left(z_G^2 + y_G^2\right)$$

$$I_{xy} = I_{x^*y^*} - mx_G y_G$$

Substituting these for the inertial tensor, I_{G} , equations (2.3 - 16) through (2.3 - 18) become:

$$\frac{\partial T_{V}}{\partial P} = I_{XX}P + I_{XY}q + I_{XZ}r$$

$$- mz_{G} (v + \dot{y}_{G})$$

$$+ my_{G} (w + \dot{z}_{G})$$

$$\frac{\partial T_{V}}{\partial q} = I_{YY}q + I_{YZ}r + I_{YZ}P$$

$$- mz_{G} (w + \dot{z}_{G})$$

$$+ mz_{G} (w + \dot{z}_{G})$$

$$+ mz_{G} (u + \dot{x}_{G})$$

$$\frac{\partial T_{V}}{\partial r} = I_{ZZ}r + I_{ZX}p + I_{ZY}q$$

$$- my_{G} (u + \dot{x}_{G})$$

$$+ mz_{G} (v + \dot{y}_{G})$$

$$(2.3 - 20)$$

Substituting these equations and equations (2.3 - 13) through (2.3 - 15) (without primed subscripts) into the Lagrangian equations of motion (2.3 - 1) through (2.3 - 6), one gets the dynamical equations:



$$\begin{split} \mathbf{X} &= \mathbf{n} \left[\stackrel{\bullet}{\mathbf{n}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} - \stackrel{\bullet}{\mathbf{x}} \left(\mathbf{q}^2 + \mathbf{r}^2 \right) + y_G \left(\mathbf{p} \mathbf{q} - \stackrel{\bullet}{\mathbf{r}} \right) + z_G \left(\mathbf{p} \mathbf{r} + \stackrel{\bullet}{\mathbf{q}} \right) \right. \\ &\quad + 2 \mathbf{q} \stackrel{\bullet}{\mathbf{x}}_G - 2 \mathbf{r} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{x}}_G \right] \\ \mathbf{Y} &= \mathbf{n} \left[\stackrel{\bullet}{\mathbf{v}} - \mathbf{p} \mathbf{w} + \mathbf{r} \mathbf{u} - y_G \left(\mathbf{r}^2 + \mathbf{p}^2 \right) + z_G \left(\mathbf{q} \mathbf{r} - \stackrel{\bullet}{\mathbf{p}} \right) + x_G \left(\mathbf{q} \mathbf{p} + \stackrel{\bullet}{\mathbf{r}} \right) \right. \\ &\quad + 2 \mathbf{r} \stackrel{\bullet}{\mathbf{x}}_G - 2 \mathbf{p} \stackrel{\bullet}{\mathbf{x}}_G + \stackrel{\bullet}{\mathbf{x}}_G \right] \\ \mathbf{Z} &= \mathbf{n} \left[\stackrel{\bullet}{\mathbf{v}} - \mathbf{q} \mathbf{u} + \mathbf{p} \mathbf{v} - z_G \left(\mathbf{p}^2 + \mathbf{q}^2 \right) + x_G \left(\mathbf{r} \mathbf{p} - \stackrel{\bullet}{\mathbf{q}} \right) + y_G \left(\mathbf{r} \mathbf{q} + \stackrel{\bullet}{\mathbf{p}} \right) \right. \\ &\quad + 2 \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G - 2 \mathbf{q} \stackrel{\bullet}{\mathbf{z}}_G + \stackrel{\bullet}{\mathbf{x}}_G \right] \\ &\quad + 2 \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G - 2 \mathbf{q} \stackrel{\bullet}{\mathbf{z}}_G + \stackrel{\bullet}{\mathbf{x}}_G \right] \\ &\quad + 2 \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_{\mathbf{z}}_G \right] \\ &\quad + \mathbf{z}_{\mathbf{x}} \stackrel{\bullet}{\mathbf{y}}_G + \mathbf{z}_{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right] \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{v}} - \mathbf{p} \mathbf{u} + \mathbf{p} \mathbf{v} + \mathbf{p} \stackrel{\bullet}{\mathbf{x}}_G + \stackrel{\bullet}{\mathbf{x}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{v}} - \mathbf{q} \mathbf{u} + \mathbf{p} \mathbf{v} + \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{v}} - \mathbf{q} \mathbf{u} + \mathbf{p} \mathbf{v} + \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{v}} - \mathbf{q} \mathbf{u} + \mathbf{p} \mathbf{v} + \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{v}} - \mathbf{q} \mathbf{u} + \mathbf{p} \mathbf{v} + \mathbf{p} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} - \mathbf{r} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} - \mathbf{r} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} - \mathbf{r} \stackrel{\bullet}{\mathbf{y}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} + \mathbf{q} \stackrel{\bullet}{\mathbf{z}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} + \mathbf{q} \stackrel{\bullet}{\mathbf{z}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} + \mathbf{q} \stackrel{\bullet}{\mathbf{z}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}{\mathbf{u}} - \mathbf{r} \mathbf{v} + \mathbf{q} \mathbf{u} + \mathbf{q} \stackrel{\bullet}{\mathbf{z}}_G + \stackrel{\bullet}{\mathbf{z}}_G \right) \\ &\quad + \mathbf{n} \mathbf{z}_G \left(\stackrel{\bullet}$$



where I, I, are about axes through the vehicle origin. These equations also contain derivatives of the form I, I, x_G , x_G

The tensor I can be thought of as being made up of two parts; a constant part representing the vehicle with all jettisonable ballast removed, I, and the jettisonable ballast, I, such that:

$$I_0 = I_1 + I_2 \tag{2.3 - 28}$$

The contribution of the ballast can be modeled by assuming that each weight that is part of the jettisemable ballast is lumped at its center of gravity (x, y, z).
gi, gi, gi.
Then:

$$I_{xx,2} = \sum_{i=1}^{N} n_i (y_{gi}^2 + z_{gi}^2)$$
 (2.3 - 29)

$$I_{xy,2} = \sum_{i=1}^{N} \max_{i \in Si} y_{gi}$$
 (2.3 - 30)

$$I_{xz,2} = \sum_{i=1}^{N} \max_{\substack{i \text{ gi gi}}} \dots \qquad (2.3 - 31)$$

where N is the number of weights.

I is the time varying part of I o, however, to be consistent with the quasi-steady approximation made in developing equations (2.3 - 7 through 2.3 - 9), a quasi-steady change in the inertial tensor must also be assumed. This then says that the terms involving time derivatives of the inertial tensor and the center of gravity can be dropped, since they are zero during the time interval over which the process is assumed to be steady.

The equations can be further reduced when it is recognized that,



due to the assumption of an xz plane of symmetry and a further assumption that the ballast will be dropped symmetrically, the products of inertia

$$I_{xy,1} = I_{yx,1} = I_{yz,1} = I_{zy,1} = I_{xy,2} = I_{yx,2} = I_{yz,2} = I_{zy,2} = 0$$

Thus, the total moment of inertia becomes quasi-steady and can be used in the equations of motion without 0, 1 or 2 subscripts. The final form of the dynamical equations then becomes:

$$X = m \left[\dot{u} - rv + qv - x_G \left(q^2 + r^2 \right) + y_G \left(pq - \dot{r} \right) + z_G \left(pr + \dot{q} \right) \right]$$
(2.3 - 32)

$$Y = m \left[\mathring{v} - p_W + ru - y_G (r^2 + p^2) + z_G (qr - \mathring{p}) + x_G (qp + \mathring{r}) \right]$$

$$(2.3 - 33)$$

$$Z = m \left[\mathring{w} - qu + pv - z_G (p^2 + q^2) + x_G (pp - \mathring{q}) + y_G (pq + \mathring{p}) \right]$$

$$(2.3 - 34)$$

$$K = I_{xx} \hat{p} + I_{xz} (\hat{r} + pq) + (I_{zz} - I_{yy}) qr$$

$$+ m \left[-z_{G} (\hat{v} - pu + ru) + y_{G} (\hat{v} + pv - qu) \right] \qquad (2.3 - 35)$$

$$M = I_{yy} d + I_{zx} (r^2 - p^2) + (I_{xx} - I_{zz}) rp$$

$$+ m \left[-x_G (t - qu + pv) + z_G (t + qv - rv) \right] \qquad (2.3 - 36)$$

$$N = I_{xx} \dot{r} + I_{xx} (\dot{p} - rq) + (I_{yy} - I_{xx}) pq$$

$$+ m \left[-y_{G} (\dot{u} - rv + qu) + x_{G} (\dot{v} + ru - pw) \right] \qquad (2.3 - 37)$$

where m, x_G , y_G , z_G , x_X , x_Z , x_Z , x_Z are all quasi-steady functions of time.

The forcing terms for the hydrodynamic equations are made up of



gravity forces, hydrodynamic forces and propulsion induced forces. Since the problem has been defined as a free ascent problem there will be no propulsion forces included. Because of this restriction, the experimental hydrodynamic coefficients should be obtained without propellers running. The total forcing terms are then:

$$X = X_G + X_H$$

$$Y = Y_G + Y_H$$

$$Z = Z_G + Z_H$$

$$K = K_G + K_H$$

$$M = M_G + M_H$$

$$N = N_G + N_H$$

$$(2.3 - 38)$$

II - 4 Gravity Forces

The hydrostatic pressure field induced by gravity creates a buoyancy force B through the CB. This force varies with ambient water density and vehicle volume. The instantaneous weight of the body, $W_{\rm a}$ acting through the CG, is made up of $W_{\rm B}$, the weight of the body without jettisonable ballast, less $W_{\rm i}$, the weight of the ballast components removed.

During the initial phase of the ascent the items in $\sum_{i=1}^{N} W_i$ are increased until all of the ballast components have been removed, at this point the buoyant force (-B + W) becomes a maximum, if we neglect changes in vehicle volume and density. This maximum force is sustained for the rest of the ascent.

The instantaneous buoyant force can thus be represented by:

$$-B + W - \sum_{i=1}^{N} V_{i}$$
 (2.4 - 1)

where N = the number of ballast components removed. This force sets



upwards along the local vertical. Due to the choice of origin of the body axes, the gravity induced torque is $\widehat{F}_G \times \widehat{B}$ where \widehat{F}_G is the CG position vector in body axes and \widehat{B} is the vehicle buoyancy vector acting up along the local vertical. Resolving along body axes the gravity forces become:

$$X_{G} = -(W - B) \sin \theta$$

$$Y_{G} = (W - B) \cos \theta \sin \phi$$

$$Z_{G} = (W - B) \cos \theta \cos \phi$$

$$Z_{G} = (W - B) \cos \theta \cos \phi$$

$$Z_{G} = (W - B) \cos \theta \cos \phi$$

$$Z_{G} = (y_{G}W - y_{B}B) \cos \theta \cos \phi - (z_{G}W - z_{B}B) \cos \theta \sin \phi$$

$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \cos \phi - (z_{G}W - z_{B}B) \sin \theta$$

$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \sin \phi + (y_{G}W - y_{B}B) \sin \theta$$

$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \sin \phi + (y_{G}W - y_{B}B) \sin \theta$$

$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \sin \phi + (y_{G}W - y_{B}B) \sin \theta$$

$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \sin \phi + (y_{G}W - y_{B}B) \sin \theta$$

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$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \sin \phi + (y_{G}W - y_{B}B) \sin \theta$$

$$Z_{G} = (x_{G}W - x_{B}B) \cos \theta \sin \phi + (y_{G}W - y_{B}B) \sin \theta$$

II - 5 Hydrodynamic Forces

The hydrodynamic forces and moments that act on a body moving through a real fluid are the result of:

- (1) hydrodynamic inertial affects (linear added mass terms),
- (2) skin friction, separation and cross-flow drag effects,
- (3) circulation offects.

These effects are all functions of the velocities and accelerations of the body. Therefore, the hydrodynamic forces and moments can be expressed functionally as:

$$\begin{bmatrix} F_{\text{H}} \\ E_{\text{H}} \end{bmatrix} = f (u, v, v, p, q, r, \hat{u}, \hat{v}, \hat{v}, \hat{p}, \hat{q}, \hat{r})$$
 (2.5 - 1)

This function may be reduced to a workable form by expanding the function in a Taylor series. Expanding the function in this form requires that the function and its derivatives be continuous in the region of the



values of the variables under consideration. A typical second order expansion of one of the force equations would be of the form:

$$X = X_{0} + u \frac{2X}{2u} + v \frac{2X}{2v} + \dots + r \frac{3X}{2r}$$

$$+ u \frac{3X}{2u} + v \frac{3X}{2v} + \dots + r \frac{3X}{2r}$$

$$+ \frac{1}{2} \left(u^{2} \frac{2X}{2u^{2}} + \sqrt{2} \frac{2X}{2v^{2}} + \dots + r^{2} \frac{2X}{2r} \right)$$

$$+ uv \frac{2X}{2u^{2}} + uv \frac{2X}{2u^{2}} + uv \frac{2X}{2u^{2}} + \dots + ur \frac{2X}{2u^{2}}$$

$$+ pq \frac{3^{2}x}{3p^{2}q} + pr \frac{2^{2}x}{3p^{2}r} + qr \frac{3^{2}x}{3q^{2}r}$$

$$+ uu \frac{3^{2}x}{2u^{2}u^{2}} + uv \frac{3^{2}x}{2u^{2}v^{2}} + \dots + ur \frac{3^{2}x}{2u^{2}r^{2}}$$

$$+ ru \frac{2^{2}x}{2u^{2}} + uv \frac{3^{2}x}{2u^{2}v^{2}} + \dots + rr \frac{3^{2}x}{2r^{2}}$$

$$+ uu \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{2v^{2}} + \dots + rr \frac{3^{2}x}{2r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{2v^{2}} + \dots + rr \frac{3^{2}x}{2r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{2v^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{2v^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{2v^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{2v^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{3u^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{3v^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{3u^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{3u^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

$$+ ru \frac{3^{2}x}{3u^{2}} + vv \frac{3^{2}x}{3u^{2}} + \dots + rr \frac{3^{2}x}{3r^{2}}$$

This equation contains ninety-one constant, linear and second order terms arising from the Taylor series expansion. The number of terms, however, can be reduced to thirty-three on the basis of the problem assumptions and restrictions delineated in Chapter I.

The constant term is dropped to conform with the requirement that



initially the only disturbance is due to a buoyant force which is included as a gravity term.

The terms involving products of accelerations with velocities or accelerations are dropped because the second order effects were restricted to velocities only. In addition the results of potential theory indicate that these derivatives are zero. (See ref 2)

After eliminating all but the linear terms and the second order velocity terms, the Taylor series expansions of the hydrodynamic forces and moments become of the form:

$$X_{H} = X_{u}u + X_{v}v + X_{u}u + X_{p}p + X_{q}q + X_{r}r$$

$$+ X_{u}u + X_{v}v + X_{u}u + X_{u}p + X_{u}p + X_{u}q + X_{v}r$$

$$+ X_{u}u^{2} + X_{u}v^{2} + X_{u}v^{2} + X_{u}v^{2} + X_{u}p^{2} + X_{u}q^{2} + X_{u}r^{2}$$

$$+ X_{u}uv + X_{u}uv + X_{u}up + X_{u}q + X_{u}r$$

$$+ X_{u}vv + X_{u}vv + X_{u}p + X_{u}q + X_{u}r$$

$$+ X_{v}vv + X_{v}pv + X_{v}qq + X_{v}r$$

$$+ X_{v}pvp + X_{u}qq + X_{v}r$$

$$+ X_{v}pq + X_{v}pr + X_{v}qr$$

$$+ X_{v}pq + X_{v}pr + X_{v}qr$$

$$(2.5 - 3)$$

The terms in this expression are seen to fall into one of three catagories, namely:

- (1) added mass or inertial,
- (2) second order non-inertial,
- (3) linear.

In order to further reduce the number of terms retained in each of these cat-



egories, it is necessary to look at the nature of the terms and the effect that a plane of symmetry has on them.

II - 5.1 Added Mass Torms

A body moving through a real fluid induces a motion in the otherwise stationary fluid because the fluid must move aside and then close in behind the body. As a result of this motion the fluid possesses kinetic energy that it would not possess if the body were not in motion. The added mass terms in the equations take into account the energy given to the fluid by the body.

If the body motion is steady, the related fluid motion is also steady which requires that the kinetic energy be constant. If the kinetic energy is constant, no work is being done on the fluid and therefore the added mass terms may be emitted.

If, however, the body is in accolerated motion, there will be work done by the body on the fluid and it will be necessary to retain at least some of the added mass terms.

Work is accomplished by moving a force through a distance. In the case of a submerged body, the distance is the distance the body travels and the force is the integral over the surface of the body of the pressures exerted by the body on the fluid. This force, in general, represents a system of forces and moments acting on the body which can be obtained from equations (2.3 - 1 through 6), when the kinetic energy is varied.

The kinetic energy of the fluid can be represented as a function of the six velocity components (u, v, v, p, q, r). A quadratic form of this function as given by Lamb (see page 172 ref 1) is:



$$2T_{f} = Au^{2} + Bv^{2} + Cv^{2} + 2A^{*}vw + 2B^{*}vu + 2C^{*}uv$$

$$+ Pp^{2} + Qq^{2} + Rr^{2} + 2P^{*}qr + 2Q^{*}rp + 2R^{*}pq$$

$$+ 2Lup + 2Hvq + 2Nur$$

$$+ 2F(vr + vq) + 2G(up + ur) + 2H(uq + vp)$$

$$+ 2F^{*}(vr - vq) + 2G^{*}(wp - ur) + 2H^{*}(uq - vp) \quad (2.5 - 4)$$

where the twenty-one coefficients A, B, C etc. are certain constants determined by the form and position of the surface relative to the co-ordinate axes.

Letting

$$F + F' = F_{1}$$
 $F - F' = F_{2}$
 $H + H' = H_{1}$ $H - H' = H_{2}$
 $G + G' = G_{1}$ $G - G' = G_{2}$

the last six terms in (2.5 - 4) become:

Nine of the coefficients in the above expression may be set to zero if we take advantage of the assumed xz plane of symmetry. Symmetry erguments say that T_f should not be changed if any of the terms vw, uv, qr, pq, up, vq, wr, vp, ur is roplaced respectively by (-v) w, (-v) u, (-r) q, (-p) q, (-p) u, (-v) q, (-r) w, (-p) w, (-r) u. These terms correspond respectively with the coefficients Λ^* , C^* , P^* , R^* , L, R', N', G, G^* .

When T_f is substituted for T_v in equations (2.3 - 1 through 6), these nine coefficients can be traced to their corresponding terms in equations (2.5 - 3), at which time these terms can be climinated from the



hydrodynamic expressions.

Six partial derivatives must be obtained from equation (2.5 - 4) in order to expand equations (2.3 - 1 through 6), these are:

$$\frac{\partial T_{f}}{\partial u} = Au + B'_{W} + C'_{V} + Lp + H_{1}q + G_{2}r$$

$$\frac{\partial T_{f}}{\partial v} = Bv + A'_{W} + C'_{u} + H'_{q} + F_{1}r + H_{2}p$$

$$\frac{\partial T_{f}}{\partial v} = Cw + A'_{V} + B'_{u} + H'_{r} + G_{1}p + F_{2}q$$

$$\frac{\partial T_{f}}{\partial v} = Pp + Q'_{r} + R'_{q} + Lu + G_{1}w + H_{2}v$$

$$\frac{\partial T_{f}}{\partial q} = Qq + P'_{r} + R'_{p} + H'_{v} + H_{1}u + F_{2}w$$

$$\frac{\partial T_{f}}{\partial r} = Rr + P'_{q} + Q'_{p} + H'_{w} + F_{1}v + G_{2}u$$

$$(2.5 - 5)$$

$$(2.5 - 6)$$

$$(2.5 - 6)$$

$$(2.5 - 7)$$

$$(2.5 - 8)$$

$$(2.5 - 9)$$

Those derivatives are then substituted in equations (2.3 - 1 through 6) to obtain:

(2.5 - 10)

$$X = A \mathring{u} + B^* \mathring{w} + C^* \mathring{v} + L \mathring{p} + H_1 \mathring{q} + G_2 r$$

$$+ C q_W + A^* vq + B^* uq + N \mathring{r} q + G_1 pq + F_2 qq$$

$$- B vr - A^* vr - C^* ur - N \mathring{q} r - F_1 rr - H_2 pr$$

$$Y = B \mathring{v} + A^* \mathring{v} + C^* \mathring{u} + M \mathring{q} + F_1 \mathring{r} + H_2 \mathring{p}$$

$$+ A ur + B^* vr + C^* vr + L pr + H_1 qr + G_2 rr$$

$$- C vp - A^* vp - B^* up - N \mathring{r} p - G_1 pp - F_2 qp$$

$$Z = C \mathring{v} + A^* \mathring{v} + B^* \mathring{u} + N \mathring{r} + G \mathring{p} + F_2 \mathring{q}$$

$$+ B vp + A^* vp + C^* up + H \mathring{q} p + F_1 rp + H_2 pp$$

$$- A uq - B^* vq - C^* vq - L pq - H_1 qq - G_2 rq$$

$$(2.5 - 13)$$



$$K = P_{P}^{h} + Q^{\dagger}\hat{r} + R^{\dagger}\hat{q} + L\hat{u} + G_{1}\hat{u} + H_{2}\hat{v}$$

$$+ Rrq + P^{\dagger}qq + Q^{\dagger}pq + Nuq + F_{1}vq + G_{2}uq$$

$$- Qqr - P^{\dagger}rr - R^{\dagger}pr - Hvr - H_{1}ur - F_{2}vr$$

$$+ Cuv + A^{\dagger}vv + B^{\dagger}uv + Nrv + G_{1}pv + F_{2}qv$$

$$- B_{vu} - A^{\dagger}vu - C^{\dagger}uw + H_{2}v - F_{1}rv - H_{2}pv$$

$$+ P_{2}rr + R^{\dagger}\hat{p} + F^{\dagger}\hat{p} + F^{\dagger}\hat{p}$$

Setting the various coefficients in equations (2.5 - 11 through 16) equal to their counterparts in equations (2.5 - 3) it is found that:



$$\Lambda^{\bullet} = X_{\text{vq}} = -X_{\text{vr}} = Y_{\bullet} = -Y_{\text{vp}} = Z_{\bullet} = Z_{\text{vp}} = K_{\text{vv}} = -K_{\text{vw}} = -H_{\text{vu}} = H_{\text{vu}}$$

$$C^{\bullet} = X_{\bullet} = -X_{uv} = Y_{\bullet} = Y_{vv} = Z_{up} = -Z_{vq} = -X_{uv} = M_{vv} = M_{uu} = -X_{vv}$$

$$G_1 = X_{pq} = -Y_{pp} = Z_p = K_s = K_p = K_{pv} = H_{pv} = -H_{pu} = -N_{vq}$$

$$G_2 = X_{\hat{\mathbf{r}}} = Y_{\mathbf{r}y} = -Z_{\mathbf{r}q} = K_{\mathbf{u}q} = -K_{\mathbf{u}p} = K_{\mathbf{r}w} = K_{\hat{\mathbf{u}}} = -K_{\hat{\mathbf{u}}} = K_{\hat{\mathbf{v}}} = -K_{\hat{\mathbf{v}}} =$$

$$L = X_{\hat{p}} = K_{\hat{u}} = H_{ur} = H_{pv} = -H_{uq} = -H_{pv}$$

$$M' = Y_{q} = -K_{qw} = M_{v} = N_{vp} = N_{qu}$$

$$N' = Z_{\hat{r}} = K_{VQ} = -M_{VP} = -N_{ru} = N_{\hat{v}}$$

$$P^* = K_{qq} = -K_{rr} = K_{r} = K_{qr} = K_{qr} = K_{rp}$$

$$R^* = K_{\mathbf{q}} = -K_{\mathbf{pr}} = M_{\mathbf{p}} = -M_{\mathbf{qp}} = N_{\mathbf{pp}} = -N_{\mathbf{qq}}$$

$$L - N' = Y_{pr} = M_{ru} = M_{pw}$$

$$M' - L = Z_{pq}$$

$$G_1 + G_2 = M_{rw} = -M_{pu}$$

$$h' - L = N_{vp} = N_{uq}$$
 (2.5 - 17)

$$H_1 + H_2 = H_{up} = - M_{vq}$$

$$B - A = N_{uv}$$
 (2.5 - 18)



$$A = X_{u} = Y_{ur} = -Z_{uq}$$

$$B = -X_{vr} = Y_{v} = Z_{vp}$$

$$C = X_{qv} = -X_{vp} = Z_{v}$$

$$B^* = X_{\mathring{\mathbf{u}}} = X_{\mathbf{uq}} = Y_{\mathbf{up}} = -Y_{\mathbf{up}} = Z_{\mathring{\mathbf{u}}} = -Z_{\mathbf{uq}} = X_{\mathbf{uv}} = X_{\mathbf{uv}} = -X_{\mathbf{uu}} = -X_{\mathbf{uv}} =$$

$$P = K_{\hat{P}} = -N_{pq}$$

$$Q = H_c = N_{qp}$$

$$Q^* = K_c = K_p = M_{pp} = M_p = N_c = -N_p$$

$$F_1 = -X_{rr} = Y_{r} = Z_{rp} = -M_{vp} = N_{v} = M_{ru}$$

$$F_2 = X_{qq} = -Y_{qp} = Z_q = M_q = -M_{qu} = N_{vp}$$

$$H_1 = X_0 = Y_{qr} = -Z_{qq} = -K_{ur} = H_0 = M_{qw} = N_{up} = -N_{qv}$$

$$H_{2} = -X_{pr} = Y_{\bullet} = Z_{pp} = K_{\bullet} = -K_{pw} = H_{vr} = -N_{vq} = N_{pu}$$

$$C - B = K$$

$$F_1 + F_2 = -K_{vr} = K_{vq}$$

$$R - Q = K_{qr}$$

$$P - R = H$$
 pr

$$A - C = H$$

$$Q - P = N_{pq}$$

(2.5 - 18) cont.



where equations (2.5 - 17) represent the terms which are zero due to symmetry.

Eliminating those terms which, due to the xz plane of symmetry, are zero, and using the standard nomenclature, as given in reference (6), in place of lamb's notation, the added mass expressions become:

$$\begin{split} X_{\text{Hi}} &= X_{\hat{\mathbf{u}}}^{\hat{\mathbf{u}}} + X_{\hat{\mathbf{u}}}^{\hat{\mathbf{u}}} + X_{\hat{\mathbf{q}}}^{\hat{\mathbf{q}}} + X_{\hat{\mathbf{q}}}^{\hat{\mathbf{q}}} + X_{\hat{\mathbf{u}}}^{\hat{\mathbf{q}}} + X_{\hat{\mathbf{u}}}^{\hat{\mathbf{q}}} \\ &+ X_{\hat{\mathbf{q}}\hat{\mathbf{q}}}^{\hat{\mathbf{q}}} + X_{\hat{\mathbf{v}}\hat{\mathbf{v}}} + X_{\hat{\mathbf{v}}}^{\hat{\mathbf{q}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{v}}}^{\hat{\mathbf{p}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} \\ &+ X_{\hat{\mathbf{q}}}^{\hat{\mathbf{q}}} + X_{\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{v}}}^{\hat{\mathbf{p}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}} + X_{\hat{\mathbf{u}}\hat{\mathbf{v}}^{\hat{\mathbf{v}}}} + X_{\hat{\mathbf$$



The equalities given in equations (2.5 - 17 and 18) are based entirely upon potential theory and do not necessarily hold in the presence of circulation and viscous effects. If circulation and viscous effects are neglected, these equalities provide reasonably good estimates of some of the second order coefficients which are not directly amenable to measurement by conventional towing tank techniques.

II - 5.2 Second Order Torms

The second order terms which are not added mass terms are:

(2.5 - 30)

 $N_{H2} = N_{2}V^{2} + N_{2}r^{2} + K_{V7}$



The symmetry plane (xz plane) force and moment terms (X, Z, H), which involve products of the symmetry plane velocities (u, u, q) with the out of plane velocities (v, p, r) must be zero for the reason that the same force or moment must result if v, p, r are replaced by v, p, r. This does not hold true with respect to products of xz plane velocities or to products of out of plane velocities.

Another symmetry argument that can be made is that a symmetry plane velocity or products of symmetry plane velocities should not cause out of the plane forces or moments. For example, there should be no Y force resulting from a w velocity or even a combination of w and u velocities.

After removing these symmetry terms from equations (2.5 - 25 through 30) the second order hydrodynamic terms left are:

$$X_{H2} = X_{u2}u^{2} + X_{v2}v^{2} + X_{u2}v^{2} + X_{p2}p^{2}$$

$$+ X_{uw}u_{w} + X_{vp}v_{p}$$
(2.5 - 31)

$$Y_{H2} = Y_{2}v^{2} + Y_{uv}uv + Y_{vw}vw + Y_{vq}vq$$
 (2.5 - 32)

$$z_{H2} = z_{u}z^{u^{2}} + z_{v}z^{v^{2}} + z_{v}z^{w^{2}} + z_{r}z^{r^{2}}$$

$$+ Z_{uw} + Z_{vr}$$
 (2.5 - 33)

$$K_{H2} = K_{p2}p^2 + K_{up}up$$
 (2.5 - 34)

$$H_{H2} = M_{2}v^{2} + H_{2}q^{2}$$
 (2.5 - 35)

$$N_{H2} = N_{x2}v^{2} + N_{yy}vy \qquad (2.5 - 36)$$



II - 5.3 Linear Terms

The linear terms in the hydrodynamic force equations are:

$$X_{HL} = X_{u} + X_{v} + X_{w} + X_{p} + X_{q} + X_{r}$$
 (2.5 - 37)

$$Y_{HL} = Y_{U} + Y_{V} + Y_{W} + Y_{D} + Y_{Q} + Y_{P}$$
 (2.5 - 38)

$$Z_{HL} = Z_{v} + Z_{v} + Z_{v} + Z_{p} + Z_{q} + Z_{r}$$
 (2.5 - 39)

$$K_{HL} = K_{U} + K_{V} + K_{W} + K_{p} + K_{q} + K_{r}$$
 (2.5 - 40)

$$N_{HL} = N_{U} + N_{V} + M_{W} + M_{p} + M_{q} + M_{r}$$
 (2.5 - 41)

$$N_{HL} = N_{u}u + N_{v}v + N_{u}v + N_{p}p + N_{q}q + N_{p}$$
 (2.5 - 42)

In order to determine which of these terms should be retained, each term must, in general be considered on its can merits. There is, however, one group of terms all of which may be eliminated on the basis of the assumed xz plane of symmetry.

The coefficients which, due to symmetry, must be zero are:

- (1) those that involve derivatives of the symmetry plane forces and moment with respect to the out of plane velocities (v, p, r).
- (2) the cut of plane force (Y) and moments (K, N) which would arise from an in the symmetry plane velocity (u, w, q).

Those that fall in the first catagory are of the type which require the force or moment to stay the same while the velocity can change sign. Those of the second category would require a force perpendicular to the plane of symmetry to result from a flow in the plane of symmetry.



Eliminating these terms from the linear terms equations, results in the following:

$$X_{HL} = X_{u}u + X_{w}u + X_{q}q \qquad (2.5 - 43)$$

$$Y_{HL} = Y_{v}v + Y_{p}p + Y_{p}r \qquad (2.5 - 44)$$

$$Z_{HL} = Z_{u}u + Z_{w}u + Z_{q}q \qquad (2.5 - 45)$$

$$K_{HL} = K_{v}v + K_{p}p + K_{p}r \qquad (2.5 - 46)$$

$$M_{HL} = M_{u}u + M_{w}u + M_{q}q \qquad (2.5 - 47)$$

$$N_{HL} = N_{v} + N_{p}p + N_{r}r$$
 (2.5 - 48)

II - 5.4 Discussion of Terms

The hydrodynamic force and moment equations that result after eliminating the symmetry terms will now be looked at in order to further reduce the number of terms in the equations.

Of first importance is the decision to retain all terms, or a form from which they can be derived, that are included in "The Standard Equations of Motion for Submarino Simulation" (see ref 8) that would apply to an unpropelled vehicle.

In general, the terms which are retained herein but are not reteined by either MSRDC or AIT/IL are done so with the idea that not all submersibles possess the near fore and aft and symmetries that modern military submarines and the DSRV possess.

The proceedure to be followed will be to look at the terms that remain after the symmetry terms have been climinated and the terms retained by other authors have been set aside.



Prior to looking at the individual force and notent quations, the effect of the choice of expansion point for the Taylor series should be investigated.

The most common operating point about which hydrodynamic forces and moments are expanded is some finite forward velocity. When this is done, the linear terms given by equations (2.5 - 43 through 48) exist. These terms include the effect of circulation which does not appear in the potential theory and the Munk moments which arise from the potential theory. Additionally these terms account for the effect that some finite initial velocity has upon the grag terms.

tial operating condition is for the vehicle to be at rest in the fluid.

Since the force causing the vehicle to ascend is the result of releasing ballast from any position on the vehicle, it is as likely for the vehicle to start moving astern as it is ahead. With this sort of operating condition, the most reasonable expansion point for the Taylor series is then the zero velocity condition. This, however, presents the problem of completely eliminating all the linear terms from the series expansion, since they must be evaluated at the expansion point, zero velocity. For example:

$$w \left[\frac{\partial v}{\partial v} \right]_{u = v_{O}} = 0 \qquad \text{since } u_{O} = 0$$

Dispite the elimination of the linear terms in the Taylor scries expansion, we are dealing with a real fluid and the effect of circualtion will still arise as the vehicle companes to move and the potential theory still indicates that the Funk noments exist. The logical place for them to be included is, of course, in the second order terms such as Many. Therefore, in the development that follows it must be remarkeded, that what is



generally included as a linear effect is now part of the second order effects.

The drag effects represented by Z_{W} , Z_{WW}^{2} , etc. in the usual case must now be represented by only the second order terms Z_{WW}^{2} , etc.

In the interest of developing as general a set of equations as possible, the linear terms usually appearing in the hydrodynamic equations shall be retained, though zero for this particular case.



II - 5.4.1 Arial Force

The axial force equation without symmetry terms is:

$$X_{H} = X_{0}u + X_{0}2q^{2} + X_{0}r^{2} + X_{0}r^{2}$$

where the first two lines represent the terms to be arbitrarily retained.

The last two lines contain those terms which require further investigation before being retained or rejected.

The linear term X_u , will be dropped in favor of the non-linear X_u^2 which equally well represents the drag phenomena. This is especially true if consideration is taken of the non-dimensionalizing parameters involved. X_u^2 is non-dimensionalized by dividing by $(\frac{1}{2}\rho l^2 U)$ where U is, in general, the velocity of the origin of the body axes. Therefore, rather than leave the dimensions of X_u^2 a function of velocity, we can take a further derivative with respect to u and eliminate the velocity dependence. This would then give us the alternative non-linear form indicated above.

The linear terms X and X are assumed to be zero on the basis of experimental results. (See table I of ref 2.)

The added mass terms X_{ij} , X_{ij} and X_{ij} are greatly dependent upon the vehicle shape. If there were fore and aft symmetry there would be no axial force resulting from X_{ij} . If, however, the vehicle had a form such as ALVAN, which possess no fore and aft symmetry, there may well be forces arising from X_{ij} . Very much similar arguments can be said for X_{ij} and X_{ij} and therefore they have been retained in the equations.



The second order term X_{uw} is assumed to be zero on the basis of experimental results (see table LTI of ref 2).

The one remaining coefficient $X_{\rm vp}$ is a second order roll transverse velocity coupling coefficient which appears to be essentially zero.

II . 5.4.2 Lateral Force

The lateral force equation without symmetry terms is:

$$Y_{H} = Y_{v}\dot{v} + Y_{v}\dot{p} + Y_{v}\dot{r} + Y_{wp} + Y_{wr} + Y_{pq}pq + Y_{qr}qr$$

$$+ Y_{v}v^{2} + Y_{vq}vq + Y_{vw}vw$$

$$+ Y_{v}v + Y_{p}p + Y_{r}r$$

$$+ Y_{ur}ur + Y_{up}p + Y_{uv}$$

$$(2.5 - 50)$$

where the first three lines represent the terms to be arbitrarily retained.

On the basis of the potential theory developed in section II \sim 5.2, Y and Y are of the order of magnitude of X, and X, respectively. Strumpf, in reference (2), retains these terms but notes that there is little or no experimental data available for Y while experimental results for Y indicate that it is important.

Similarly, Y_{uv} is also believed to be of importance on the basis of experimental results cited in reference (2).

II - 5.4.3 Normal Force

The normal force equation without symmetry terms is:



$$Z_{H} = Z_{v}\dot{u} + Z_{v}\dot{q} + Z_{v}vp + Z_{rp} + Z_{pp}^{2}$$

$$+ Z_{vv}v^{2} + Z_{vw}^{2} + Z_{rr}^{2} + Z_{vr}vr$$

$$+ Z_{v}v^{2} + Z_{q}q + Z_{u}u$$

$$+ Z_{v}\dot{u} + Z_{q}q + Z_{u}u$$

$$+ Z_{v}\dot{u} + Z_{u}q + Z_{vq} + Z_{q}q + Z_{u}u^{2} + Z_{uw}^{2}$$

$$(2.5 - 51)$$

where the first three lines are the arbitrarily retained terms.

The added mass terms $Z_{\hat{u}}$, $Z_{\hat{u}q}$, $Z_{\hat{u}q}$ and $Z_{\hat{q}q}$ are retained on the possibility of a less symmetric vehicle's giving rise to this sort of term. Potential theory estimates $Z_{\hat{u}q}$ to be of order $X_{\hat{u}}$, which is not negligible, $Z_{\hat{u}}$ and $Z_{\hat{u}q}$ to be of order $X_{\hat{u}}$, which was not neglected previously and $Z_{\hat{q}q}$ of order $X_{\hat{u}}$ according to potential theory.

The $Z_{\mbox{uu}}$ term is shown to be important on the basis of emperimental results (see ref 2).

II - 5,4,4 Rolling Moment

The rolling moment equation without symmetry terms is:

$$K_{H} = K_{\bullet} \dot{p} + K_{\bullet} \dot{r} + K_{\bullet} \dot{v} + K_{rq} rq + K_{pq} pq + K_{vq} vq$$

$$+ K_{vr} vr + K_{vv} vv + K_{vp} vp + K_{p2} p^{2}$$

$$+ K_{v} v + K_{p} p + K_{r}$$

$$+ K_{v} vv + K_{ur} \dot{v}r + K_{up} up \qquad (2.5 = 52)$$

where the first three lines represent the arbitrarily retained terms.

Here again there is little or no experimental evidence available from which estimates of $K_{{\bf u}{\bf v}}$, $K_{{\bf u}{\bf v}}$ and $K_{{\bf u}{\bf p}}$ may be made. Potential theory indicates that these terms are small and, therefore, those terms are neglected.



II - 5.4.5 Pitching Mount

The pitching moment equation without symmetry terms is:

$$H_{H} = H_{\bullet}\dot{q} + H_{\bullet}\dot{w} + H_{rr}^{2} + H_{vr} + H_{rp}^{2} + H_{pp}^{2} + H_{pp}^{2}$$

$$+ H_{vv}^{2} + H_{vq} + H_{uv}^{2}$$

$$+ H_{vv}^{2} + H_{qq}^{2} + H_{v} + H_{q} + H_{u}$$

$$+ H_{vv}^{2} + H_{qq}^{2} + H_{v} + H_{q} + H_{u}$$

$$+ H_{vv}^{4} + H_{uv}^{4} + H_{uq}^{4} + H_{uq} + H_{uq}$$

$$(2.5 - 53)$$

where the first three lines contain the arbitrarily retained terms.

Equations (2.5 - 18) show that the added mass term H_{u} is of the same size as X, and M which have been retained. Similarly H is of the q qw same size as M.. Experimental evidence cited by Strumpf supports the retention of H .

The term M is, in accordance with equations (2.5 - 18), equal to $(X_u - Z_v)$. For most vehicles the Z, term is considerably larger than X_v and therefore the term M should be retained.

II - 5.4.6 Yawing Moment

The yawing moment equation without symmetry terms is:



where the first four lines contain the terms arbitrarily retained.

Equations (2.5 - 18) show that N_v is equal to $Y_v - X_v$, $N_v = N_v$ and $N_v = N_v$. Since Y_v is of order of magnitude larger than X_v , N_v is of order of magnitude larger than X_v , N_v is of order Y_v and not negligible.

No and N_v and rotained by the other authors and therefore, N_v and N_v and N_v and N_v and N_v are should also be retained in order to be consistent.

II - 6 Equations of Motion for Free Ascent

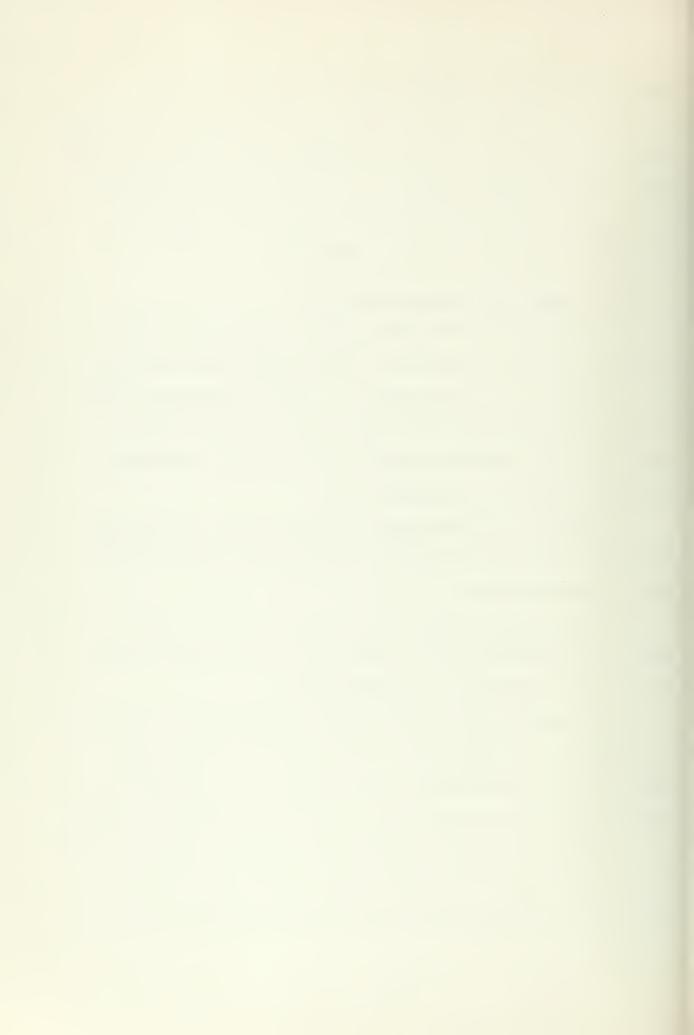
The conservation of momentum equations, equations (2.3 - 32 through 37), as derived in section II - 3, the gravity forces, equations (2.4 - 2 through 2.4 - 7) from section II - 4 and the hydrodynamic force equations developed in section II - 5 will now be combined to give the final form for the equations of motion for subhersibles in six degrees of freedom with varying mass and center of mass.

Several of the terms retained are not generally found in most developments because the terms are not experimentally or analytically obtained by present methods.

As it is not the purpose of this paper to evaluate additional terms for the equations, these additional terms will be retained but set to zero in the computation of ascent trajectories.

The equations of motion for a vehicle with varying mass will be presented in the following manner: the left hand side of the equation represents the rigid body dynamics, the right hand side represents the hydrodynamic forces and moments acting on the body and causing the motions.

The hydrodynamic terms presented here have been non-dimensionalized in the usual manner as described in reference (6). Typical non-dimensional forms of the hydrodynamic terms are presented in the non-nelature.



For simplicity, the primes have been omitted from the term in the equations presented.

The equations of motion for a freely ascending vehicle with variable mass are:

AXIAL FORCE

LATERAL FORCE

$$\frac{f}{2} 1^{2} (Y_{vv}v^{2} + Y_{vuv} + Y_{vv}uv + Y_{vv}vw) \\
+ \frac{f}{2} 1^{3} (Y_{v}\dot{v}^{2} + Y_{uv}uv + Y_{vu}uv + Y_{vv}vw) \\
+ \frac{f}{2} 1^{3} (Y_{v}\dot{v} + Y_{uv}uv + Y_{vu}uv + Y_{vv}vq) \\
+ \frac{f}{2} 1^{3} (Y_{uv}uv + Y_{vu}uv + Y_{vv}vq) \\
+ \frac{f}{2} 1^{3} (Y_{uv}uv + Y_{vv}uv + Y_{vv}vq) \\
+ \frac{f}{2} 1^{4} (Y_{v}\dot{v} + Y_{v}\dot{v} + Y_{vv}vq) \\
+ \frac{f}{2} 1^{4} (Y_{v}\dot{v} + Y_{v}\dot{v} + Y_{vv}vq) \\
+ (W - B) \sin \phi \cos \theta \qquad (2.6 - 2)$$



NORMAL FORCE

$$\frac{f}{2} x^{2} \left(z_{u}uu + z_{u}u^{2} + z_{v}v^{2} + z_{u}u^{2} + z_{u}uu + z_{u}uw \right) + \frac{f}{2} x^{2} \left(z_{u}uu + z_{u}u^{2} + z_{v}v^{2} + z_{u}u^{2} + z_{u}uu + z_{u}uw \right) + \frac{f}{2} x^{3} \left(z_{u}u + z_{u}u^{2} + z_{v}vv + z_{u}vv + z_{u}uu + z_{u}uw \right) + \frac{f}{2} x^{4} \left(z_{u}u + z_{u}v^{2} + z_{v}vv + z_{u}vv + z_{u}uu + z_{u}vv \right) + \frac{f}{2} x^{4} \left(z_{u}u + z_{u}v^{2} + z_{u}v^{2} + z_{u}v^{2} + z_{u}v^{2} + z_{u}v^{2} \right) + (w - b) \cos \theta \cos \phi \qquad (2.6 - 3)$$

ROLLING MOMENT

$$I_{xx}\dot{p} + I_{xz} (f + pq) + (I_{zz} - I_{yy}) qr$$

$$+ m \left[y_{G} (\dot{v} + pv - qu) - z_{G} (\dot{v} + ru - pv) \right] =$$

$$-\frac{\rho}{2} 1^{3} (K_{vv}v^{2} + K_{vuv} + K_{vvv})$$

$$+ \frac{\rho}{2} 1^{4} (K_{\dot{v}}\dot{v} + K_{up} + K_{ur} + K_{vq}vq + K_{up} + K_{ur})$$

$$+ \frac{\rho}{2} 1^{5} (K_{\dot{p}}\dot{p} + K_{\dot{r}}\dot{r} + K_{pp}p^{2} + K_{pq}pq + K_{qr}qr)$$

$$+ (y_{G}W - y_{B}B) \cos \theta \cos \phi - (z_{G}W - z_{B}B) \cos \theta \sin \phi$$

$$(2.6 - 4)$$



PUTCHING MOMENT

$$I_{yy} \dot{q} + I_{xz} (r^{2} + p^{2}) + (I_{xx} - I_{zz}) rp$$

$$+ m \left[z_{G} (\dot{u} + q_{W} - rv) - x_{G} (\dot{u} + p_{W} - qu) \right] =$$

$$- \frac{\ell}{2} 1^{3} (H_{uu} u^{2} + H_{u} u^{2} + H_{vv} v^{2} + H_{uw} u^{2} + H_{uw} u + H_{uu})$$

$$+ \frac{\ell}{2} 1^{4} (H_{u}\dot{u} + H_{u}\dot{u} + H_{vp} vp + H_{vr} vr + H_{wq} vq)$$

$$+ \frac{\ell}{2} 1^{4} (H_{u}uq + H_{uq}uq)$$

$$+ \frac{\ell}{2} 1^{5} (H_{u}\dot{q} + H_{pp}p^{2} + H_{qq}q^{2} + H_{rr}r^{2} + H_{rp}rp)$$

$$- (x_{G}W - x_{B}B) \cos \theta \cos \phi - (z_{G}W - z_{B}B) \sin \theta \qquad (2.6 - 5)$$

YAWING LOLENT

$$\mathbf{I}_{zz} \dot{\mathbf{r}} + \mathbf{I}_{xz} (\dot{\mathbf{p}} + \mathbf{rq}) + (\mathbf{I}_{yy} - \mathbf{I}_{xx}) pq
+ m \left[\mathbf{x}_{G} (\dot{\mathbf{v}} + \mathbf{ru} - \mathbf{pw}) - \mathbf{y}_{G} (\dot{\mathbf{u}} + \mathbf{qw} - \mathbf{rv}) \right] =
- \frac{P}{2} \mathbf{1}^{3} (\mathbf{N}_{vv} \mathbf{v}^{2} + \mathbf{N}_{vuv} + \mathbf{N}_{uv} \mathbf{uv} + \mathbf{N}_{vw} \mathbf{vw}
+ \frac{P}{2} \mathbf{1}^{4} (\mathbf{N}_{v} \dot{\mathbf{v}} + \mathbf{N}_{vuv} + \mathbf{N}_{uv} \mathbf{uv} + \mathbf{N}_{vw} \mathbf{uv} + \mathbf{N}_{uv} \mathbf{uv})
+ \frac{P}{2} \mathbf{1}^{4} (\mathbf{N}_{up} \mathbf{up} + \mathbf{N}_{vu} \mathbf{uv} + \mathbf{N}_{vq} \mathbf{vq})
+ \frac{P}{2} \mathbf{1}^{5} (\mathbf{N}_{p} \dot{\mathbf{p}} + \mathbf{N}_{v} \dot{\mathbf{r}} + \mathbf{N}_{rr} \mathbf{r}^{2} + \mathbf{N}_{pq} \mathbf{pq} + \mathbf{N}_{qr} \mathbf{qr}
+ (\mathbf{x}_{c} \mathbf{W} - \mathbf{x}_{b} \mathbf{B}) \cos \theta \sin \phi + (\mathbf{y}_{c} \mathbf{W} - \mathbf{y}_{c} \mathbf{B}) \sin \theta \qquad (2.6 - 6)$$



III - 7 Sumary

The axis systems to be used in the problem are discussed and the transformations from the earth fixed to the body fixed axes are developed.

A derivation of the dynamical equations for a vehicle with a varying mass and center of mass is then made using a Lagrangian formalism. An
assumption that the reduction in mass due to releasing ballast can be represented as a quasi-steady process is discussed.

The forces acting on the body are then developed by expanding a functional representation of the forces in a Taylor series. The terms in the series are then discussed.

The equations are then non-dimensionalized for final presentation.



CHAPTER ILI

SOLUTION OF THE EQUATIONS OF ACTION

In the previous chapter a system of equations of notion were developed for which a solution must be found in order to obtain the ascent trajectory of a vehicle. It is possible that an analytic solution to this set of simultaneous non-linear differential equations could be found, however, it is much more practical to assume that a stepuise linear approximation to these equations. This approach has been used in many simulation studies of which the DSRV control system simulation (see ref 5) and determining the effect of hull shape non-linearities (see ref 4) are just two.

used by Strumpf (see ref 2) which does not conform with either the hydrodynamic terms format used by the EIT Instrumentation Laboratory for the DSRV control system studies and simulation (see ref 5), or the standard equations of motion for submerine simulation as used by NSRDC (see ref 8). In order that available hydrodynamic coefficients be utilized, the notation of Chapter II will be modified in this chapter to conform with that of the NSRDC equations. The choice of the NSRDC form over the MIT/IL form was made for the reason that it is more likely that vehicle coefficients will be obtained from NSRDC than from MIT/IL which must also get its data from other sources. The MIT/IL notation does possess the advantage of having the dimensional forms of the coefficients independent of velocity.

Before a stepwise solution to the equations can be obtained, the equations must be put into a form which is appenable to this technique.



III - 1 Revised Equations of Motion

The vehicle equations of motion are generally expressed in the form:

$$n\frac{d\vec{v}}{dt} = -n\left(\frac{d\vec{l}}{dt} \times \vec{D}\hat{i} + \vec{V} \times (\vec{v} + \vec{v} \times \vec{D}\hat{i})\right) + \vec{F}_{HYD} + \vec{F}_{EFF} \quad (3.1 - 1)$$

$$\ddot{1} \frac{d\vec{\hat{W}}}{dt} = -\vec{\hat{W}} \times \vec{\hat{W}} - m\vec{\hat{B}}\vec{\hat{G}} \times (\frac{d\vec{\hat{V}}}{dt} + \vec{\hat{W}} \times \vec{\hat{V}}) + \vec{\hat{H}}_{HYD} + \vec{\hat{H}}_{EFF}$$
 (3.1 - 2)

where $\frac{d\widetilde{V}}{dt} = \mathring{u}$, \mathring{v} , \mathring{v} , $\frac{d\widetilde{W}}{dt} = \mathring{p}$, \mathring{q} , \mathring{r} and \mathring{I} is the inertial tensor.

To determine ascent trajectories in a stepwise linear fashion on a digital computer it is useful to rearrange the above equations so that all the derivatives are on the left side of the equations. The resulting equations are of the form:

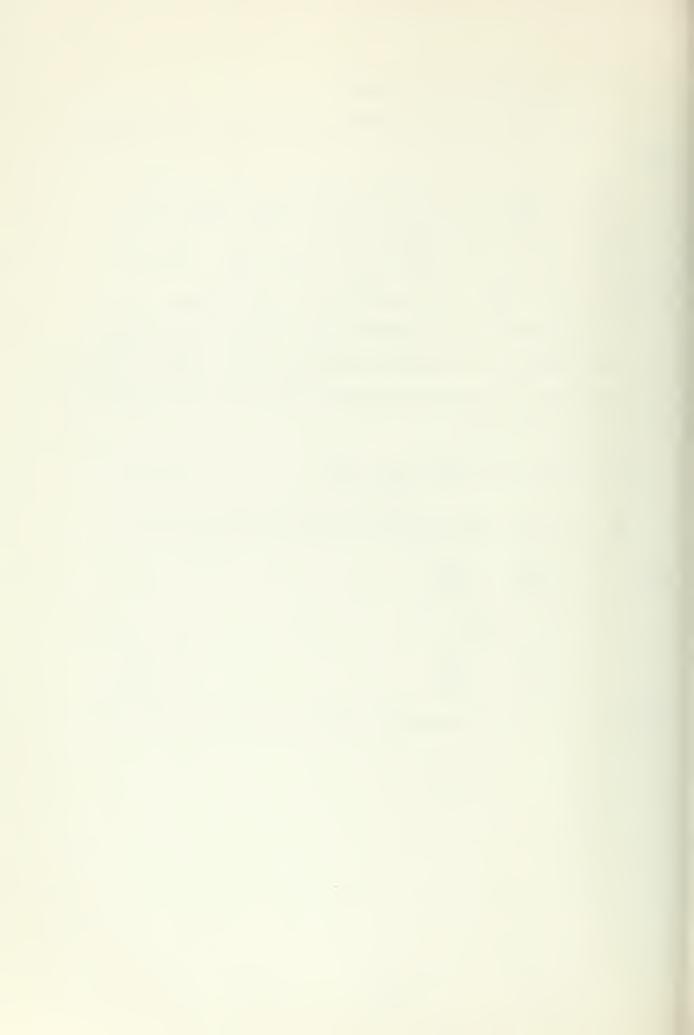
$$\frac{d\hat{V}'}{dt} = -m\hat{W} \times (\hat{V} + \hat{W} \times \hat{EG}) + \hat{F}_{HYD} + \hat{F}_{EFF}$$
 (3.1 - 3)

$$\frac{d\vec{V}}{dt} = -\vec{V} \times \vec{W} - \vec{m}\vec{E}\vec{G} \times (\vec{V} \times \vec{V}) + \vec{E}\vec{G} \times \vec{V}_{V} + \vec{H}_{HYD} + \vec{M}_{EFF}$$
(3.1 - 4)

ocentu

$$\begin{bmatrix} \frac{d\hat{V}}{dt} \\ \frac{d\hat{V}}{dt} \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} \frac{d\hat{V}}{dt} \\ \frac{d\hat{V}}{dt} \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} \frac{d\hat{V}}{dt} \\ \frac{d\hat{V}}{dt} \end{bmatrix}$$
(3.1 - 5)

and [H] is the six by six derivative coefficient matrix given on page 48.



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The actual derivatives $\frac{dV}{dt}$ and $\frac{dV}{dt}$ are obtained from

$$\begin{bmatrix} \frac{\partial \hat{V}}{\partial t} \\ \frac{\partial \hat{V}}{\partial t} \end{bmatrix} = \begin{bmatrix} A \\ C \\ C \end{bmatrix} = \begin{bmatrix} A \\ C \\ C \end{bmatrix} = \begin{bmatrix} A \\ C \\ C \end{bmatrix}$$
(3.1 - 6)

where $\begin{bmatrix} M \end{bmatrix}^{-1}$ is the inverse of $\begin{bmatrix} M \end{bmatrix}$.

The equations to be solved for $\frac{dV}{dt}$ and $\frac{dJ}{dt}$ are then given by the following:

$$\frac{d\mathbf{n}'}{d\mathbf{t}} = - \operatorname{m} \left[- \operatorname{rv} + \operatorname{qu} - \mathbf{x}_{G} \left(\mathbf{q}^{2} + \mathbf{r}^{2} \right) + \mathbf{y}_{G} \operatorname{pq} + \mathbf{z}_{G} \operatorname{pr} \right]$$

$$+ \frac{f}{2} \mathbf{1}^{2} \left(\mathbf{x}_{uu} \mathbf{u}^{2} + \mathbf{x}_{vv} \mathbf{v}^{2} + \mathbf{x}_{uv} \mathbf{u}^{2} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{3} \left(\mathbf{x}_{uq} \mathbf{u} + \mathbf{x}_{vr} \mathbf{v} + \mathbf{x}_{uq} \mathbf{u} \mathbf{q} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{4} \left(\mathbf{x}_{qq} \mathbf{q}^{2} + \mathbf{x}_{rr} \mathbf{r}^{2} + \mathbf{x}_{rp} \operatorname{rp} \right)$$

$$+ \left(\mathbf{w} - \mathbf{B} \right) \sin \theta + \mathbf{x}_{EFF}$$

$$+ \frac{f}{2} \mathbf{1}^{2} \left(\mathbf{y}_{v|v} \mathbf{v} + \mathbf{v}_{v} \mathbf{v}^{2} + \mathbf{v}^{2} \right) + \mathbf{z}_{G} \mathbf{q} \mathbf{r} + \mathbf{x}_{G} \mathbf{q} \mathbf{p} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{2} \left(\mathbf{y}_{v|v} \mathbf{v} + \mathbf{v}_{v} \mathbf{v}^{2} + \mathbf{v}^{2} \right) + \mathbf{v}_{v} \mathbf{v} \mathbf{v} + \mathbf{v}_{v} \mathbf{v}^{2} + \mathbf{v}_{v} \mathbf{v}^{2} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{3} \left(\mathbf{y}_{up} + \mathbf{y}_{rur} + \mathbf{y}_{vq} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v}^{2} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{3} \left(\mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{3} \left(\mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{3} \left(\mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} + \mathbf{y}_{v} \mathbf{v} \right)$$

$$+ \frac{f}{2} \mathbf{1}^{4} \left(\mathbf{y}_{v} \mathbf{p} \mathbf{q} + \mathbf{y}_{q} \mathbf{q} \mathbf{r} \right) + \left(\mathbf{w} - \mathbf{B} \right) \sin \phi \cos \theta + \mathbf{y}_{EFF}$$

$$(3.1 - 8)$$



$$\begin{aligned} \frac{\partial r^{4}}{\partial t} &= -m \left[-qu + pv - z_{G} \left(p^{2} + q^{2} \right) + z_{G}rp + y_{G}rq \right] \\ &+ \frac{\rho}{2} \, 1^{2} \left(Z_{u}uu + Z_{vv}v^{2} + Z_{u|v|}u \, | \left(v^{2} + v^{2} \right)^{\frac{1}{2}} \right| + Z_{u}uv \right) \\ &+ \frac{\rho}{2} \, 1^{2} \left(Z_{|v|}u|v \, | + Z_{vv}u \, | \left(v^{2} + v^{2} \right)^{\frac{1}{2}} \right| \right) \\ &+ \frac{\rho}{2} \, 1^{3} \left(Z_{vp}vp + Z_{vr}vr + Z_{q}uq + Z_{v|q} \right) \left[\left(v^{2} + v^{2} \right)^{\frac{1}{2}} \right] |q| \right) \\ &+ \frac{\rho}{2} \, 1^{4} \left(Z_{pp}p^{2} + Z_{qq}q^{2} + Z_{rr}r^{2} + Z_{rp}rp \right) \\ &+ \left(W - B \right) \cos \theta \cos \phi + Z_{EFF} \end{aligned} \qquad (3.1 - 9) \\ \frac{dp^{4}}{dt} &= -1 \sum_{XZ}pq + \left(\sum_{YY} - \sum_{ZZ} \right) qr - n \left[-z_{G} \left(-pv + rq \right) + y_{G} \left(pv - qu \right) \right] \\ &+ \frac{\rho}{2} \, 1^{3} \left(K_{v|v|}v \, | \left(v^{2} + v^{2} \right)^{\frac{1}{2}} \right] + K_{vuv} + K_{vvvu} + K_{vu}^{2} \right) \\ &+ \frac{\rho}{2} \, 1^{4} \left(K_{pup} + K_{rur} + K_{vq}vq + K_{vu}v + K_{vvvu} + K_{vvv}^{2} \right) \\ &+ \frac{\rho}{2} \, 1^{5} \left(K_{p|p} \, | p|p \, | p \, | + K_{pq}^{2} pq + K_{qr}^{2} \right) \\ &+ \left(y_{G}^{2} V - y_{B}^{2} \right) \cos \theta \cos \phi - \left(z_{G}^{2} V - z_{B}^{2} \right) \cos \theta \sin \phi + K_{EFF}^{2} \end{aligned}$$



$$\frac{dq^{4}}{dt} = -I_{XZ} \left(r^{2} - p^{2}\right) + \left(I_{ZZ} - I_{ZX}\right) rp$$

$$- m \left[-x_{G} \left(-qu + pv\right) + I_{G} \left(qv - zv\right)\right]$$

$$+ \frac{p}{2} I^{3} \left(I_{1} u^{2} + I_{1} v^{2} + I_{1} u_{1} u^{1} u^{1} \left[\left(v^{2} + v^{2}\right)^{\frac{3}{2}}\right] + I_{1} uv_{1}\right)$$

$$+ \frac{p}{2} I^{3} \left(I_{1} u^{2} + I_{1} v^{2} + I_{2} u^{2}\right)^{\frac{3}{2}} + I_{1} u^{1} u^{1} u^{1}\right)$$

$$+ \frac{p}{2} I^{3} \left(I_{1} u^{1} u^{2} + I_{2} v^{2} + I_{2} u^{2} + I_{1} u^{1} u^{1}\right)$$

$$+ \frac{p}{2} I^{3} \left(I_{1} v_{1} v^{2} + I_{2} v^{2} + I_{2} u^{2} + I_{1} u^{2}\right)$$

$$+ \frac{p}{2} I^{5} \left(I_{1} p_{2} v^{2} + I_{1} u^{2}\right) q^{1} q^{1} q^{1} + I_{1} v^{2} x^{2} + I_{1} rp$$

$$- \left(x_{0} v - x_{0} v^{2}\right) \cos \theta \cos \phi - \left(x_{0} u - x_{0} v^{2}\right) \sin \theta + I_{EFF}$$

$$\left(3.1 - 11\right)$$

$$\frac{dr}{dt} = -I_{ZX} rq + \left(I_{ZX} - I_{YY}\right) pq - n \left[-y_{G} \left(-rv + qu\right) + y_{G} \left(ru - pu\right)\right]$$

$$+ \frac{p}{2} I^{3} \left(I_{1} v_{1} v^{2} + v^{2}\right)^{\frac{3}{2}} + I_{1} u_{2} v_{3} + I_{2} u^{2}$$

$$+ \frac{p}{2} I^{4} \left(I_{1} u_{2} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2}$$

$$+ \frac{p}{2} I^{4} \left(I_{1} u_{2} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2}$$

$$+ \frac{p}{2} I^{4} \left(I_{1} u_{2} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2}$$

$$+ \frac{p}{2} I^{5} \left(I_{1} v_{1} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2}$$

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$$+ \frac{p}{2} I^{5} \left(I_{1} u_{2} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2}$$

$$+ \frac{p}{2} I^{5} \left(I_{1} u_{2} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2} u^{2}$$

$$+ \frac{p}{2} I^{5} \left(I_{1} u_{2} v^{2} + I_{2} u^{2}\right) + I_{2} u^{2} u^{2}$$

$$+ \frac{p}{2} I^{5} \left(I_{1} u_{2} v^{2} + I_{2} u^$$

In those equations, unlike equations (2.6 ~ 1 through 7) from which they come, the linear terms Y_v , Y_p , Y_r , Z_u , Z_v , H_v , $H_$



inseparable from a model test data reduction vic. point. The terms $Z_{|V|}$, $Z_{|V|}$, $Z_{|V|}$ and $Z_{|V|}$ have been added so as to include all of the NSHDC coefficients. The terms $Z_{\rm EFF}$, $Z_{\rm EFF}$, and $Z_{\rm EFF}$ have been included in order to allow for the possibility of there being some small effector forces on the vehicle. This inclusion in no way affects the legitimacy of the equations so long as the force does not cause the predominance of one velocity component.

With the equations in this form, a stepwise linear solution can be developed which will be capable of being programmed for use on a high speed digital computer.

III - 2 Stepwise Linear Solution

A steprise linear solution is one in which the accelerations are taken as constant over a given time interval. The accelerations that exist over a time interval are determined from the velocities that existed at the end of the previous time interval and the total weight removed from the vehicle. Thus, the right hand side of equations (3.1 - 3 and 3.1 - 4) can be determined at the beginning of each time step. The inertia terms in [4] must also be recomputed before each step since the mass is changing in a stepwise linear fashion as described in section II - 3. With the foregoing information equation (3.1 - 6) can be solved for the body axes accelerations.

The velocity of the vehicle in the body axis system may then be found by:

$$\begin{bmatrix} \hat{V} \\ \hat{V} \end{bmatrix}_{t} = \begin{bmatrix} \hat{V} \\ \hat{V} \end{bmatrix} dt$$

$$(3.2 - 1)$$



where to is the time at which the ascent started and t is the present time.

The integral may, however, be represented as a sum of integrals over each step in the stepmise solution. Thus:

$$\begin{bmatrix} \hat{V} \\ \hat{V} \end{bmatrix} = \sum_{n=1}^{N} t_n \begin{bmatrix} \hat{V} \\ \hat{V} \end{bmatrix} t_n = 1$$
 dt (3.2 - 2)

where t_{n-1} and t_n are respectively the times at the beginning and end of the interval. The solution has been specified as stepwise linear and the accelerations are to be obtained as constants for the duration of an interval, therefore, the accelerations can be removed from the integral leaving only the trivial integration of dt from t_{n-1} to t_n . Defining At as $t_n - t_{n-1}$: equation (3.2 - 2) becomes:

$$\begin{bmatrix} \overrightarrow{V} \\ \overrightarrow{V} \end{bmatrix}_{t_n} = \begin{bmatrix} \overrightarrow{V} \\ \overrightarrow{V} \end{bmatrix}_{t_{n-1}}$$
 Δt (3.2 - 3)

which gives the translational and rotational velocities of the vehicle in the body axis system.

In order to obtain the actual position of the vehicle relative to its inertial starting point the velocities obtained above must be transformed into the inertial axis system by use of the inverse of the transformation matrixes developed in section II = 2.3. The velocities \mathring{x} , \mathring{y} , \mathring{z} , $\mathring{\phi}$, $\mathring{\psi}$ are obtained from:



$$\begin{bmatrix} \dot{x}_{E} \\ \dot{y}_{E} \\ \dot{z}_{E} \end{bmatrix} = T_{B}^{-1} \begin{bmatrix} u \\ v \end{bmatrix}; \begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = A^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$(3.2 - 4);$$

$$(3.2 - 5)$$

The position and orientation in the inertial axis system is then determined by integrating the respective volocities from t_0 to t. Here again the solution is stepwise and may be represented as a sum of the individual steps. In this case, however, the integrand \dot{x}_p , \dot{y}_p , etc. is not a constant. It is, instead, a linear function of time since the acceleration from whence it came is a constant. Therefore the equation describing the vehicle trajectory and orientation becomes:

whore $x_E = (x_E, y_E, z_E)$ and $\phi = (\phi, \theta, \psi)$.

The computation of \mathbf{x}_E and ϕ are the final computations of the step. The procedure is then repeated until some maximum time is reached or some predefined position is reached.

In order to verify the result obtained by the foregoing procedure, and independent trajectory determination must be made.

III - 3 One Dimensional Ascent Trajectory

Now that a three dimensional second order method of determining free ascent trajectories has been obtained, some check on the results of this procedure is in order. The most logical approach is to reduce the problem to one in the vertical plane only and thereby reducing the complexity of the problem. If a derivation of the equations of motion were



made at this point the resulting equation, would be no different from those previously obtained except that some of the coefficients would now be zero. In addition, the solution of the two dimensional equations for a complete trajectory would require the aid of a high speed digital computer.

To avoid the necessity of employing a computer, a simplified method described by Giddings and Louis in reference (9) will be used. This method, as applied to the problem under consideration, reduces to a one dimensional solution of the equations of motion for a vehicle with varying mass.

The one dimensional solution determines the vertical position, velocity and acceleration of the vehicle during its ascent. This information provides a sufficient check of the results of the computer solution of the equations of motion developed in Chapter II. A description of the one dimensional solution follows.

Defining:

B - the net buoyant force

b - the time rate of change of B

F - the sum of all forces in the vertical direction

g - the acceleration due to gravity

k - the virtual mass coefficient

m' - the mass of the vehicle plus the virtual mass

W - the instantaneous vehicle weight

z, ż, ż - the vertical displacement, velocity and acceleration respec-

 $Z_{_{
m UW}},~Z_{_{
m U}}$ - the crossflow drag and added mass coefficients respectively

Newtons lass of motion can be empressed as:



$$F = m'^{\circ}_{Z} \tag{3.3 - 1}$$

The force F is made up of the weight of the jettisenned ballast, B, plus or minus the hull drag, D, depending on the direction of metion.

The force due to the jettisenning of ballast is given by:

$$B = \int_{0}^{t} b(t) dt$$
 (3.3 - 2)

Letting the integral be represented by a series of finite steps the buoyant force becomes:

$$B = \sum_{n=1}^{N} b_{n}$$
 (3.3 - 3)

where the b are a sequence of finite weights to be jettisonmed and n is the number of intervals ellapsed since time zero.

The force due to hull drag is represented by:

$$D = \frac{1}{2} \rho \, 1^2 \, Z \, \frac{\dot{z}}{z} |\dot{z}| \tag{3.3 - 4}$$

The mass, m', of the vehicle can be written as:

$$\mathbf{m'} = \frac{\mathbf{W}}{\mathbf{g}} \mathbf{k} \tag{3.3 - 5}$$

where
$$k = \left[Z_{*} \left(\frac{1}{2} / 2 \right)^{3} \right) / (W/8) \right] + 1$$
 (3.3 - 6)

Substituting these expressions into equation (3.3 - 1) the equation of motion becomes:



$$(z_{\nu}(\frac{1}{2}P1^{3}) + \frac{11}{8}) = z_{\nu}(\frac{1}{2}P1^{2}) = \sum_{n=1}^{N} b_{n} = 0$$
 (3.3 - 7)

The technique of step by step integration similar to that used in section III - 2 may now be applied to this equation in order to obtain the one dimensional trajectory.

III - 4 Summary

The notation and terms format to be used in the computer simulation is discussed. The notation used in the equations of motion is modified to conform with that of NSRDC and the equations are rearranged for digital computer solution.

A solution method is developed utilizing a steprise linear technique. It accepts the vehicle velocities, position, orientation and buoyancy as initial conditions and using equations (3.1 - 6 through 3.1 - 12)
it computes the accelerations, velocities, and displacements of the vehicle
after a time interval At. Before each step the weight, CG, mass and moments
of inertia are adjusted.

Finally a simplified one dimensional method is devised for comparison with the computer results.



CHAPTIM IV

RESULTS AND COLCLUSIONS

In Chapter III the equations of motion were derived for a freely ascending vehicle with varying mass and center of mace. A stepwise linear solution suitable for programming on a digital computer was developed in Chapter III, and the actual program is presented in Appendix A. An equation for a one dimensional ascent trajectory was also developed in Chapter III to provide a check for the three dimensional, six degree of freedom, ascent trajectories program. A program to solve this non-linear differential equation is presented in Appendix D.

In this chaater, the computer simulations and results of these simulations are discussed, conclusions drawn and recommendations for future work in this area made.

IV - 1 Results of Computer Simulations

Computer simulations of the ascent trajectories for a vehicle similar to the Deep Submergence Rescue Vehicle were conducted using the IEM 360 computer of the EET Information Processing Center. These simulations were conducted primarily for the purpose of debugging the ascent trajectories program and determining its operating characteristics and secondarily to study the motion tendancies of the DSRV.

IV - 1.1 Program Operating Characteristics

The ascent trajectories program in calculating the vehicle trajestionies depends upon the assumed time increment for its accuracy. In general, the time increment assumed can be chessed as either too large, too small or all right. Note that the latter category was "all right" in t "just in ht".



The matter of just the right increment requires a discussion of the factors influencing the situation and, therefore, the former categories will be dealt with first.

Should the time increment chosen be too large, the forces acting on the volicle will not be damped in a natural fashion. A vehicle under the influence of a steady force will continue to accelerate until this force is overcome by another force, and the computer does not see another force until the present time increment ends and the forces resulting from the present motion can be calculated. If the time increment is too long, the motions calculated by the computer become excessive. The eventual result is that the computer run is prematurely terminated. The actual cause of termination can be either program caused or machine caused.

A run is terminated by the program if either the time limit or the depth limit is exceeded. Since reaching the time limit or reaching the depth limit in a natural fashion do not constitute being premature, they are not considered here. The depth limit can, however, be reached in an unnatural fashion as is indicated in Appendix C, section C - 3.

A run is terminated by the computer if the number of extreme value calculations becomes excessive.

Termination for the latter cause occurs before the normal output can be made and, therefore, the only printed output received till be the initial condition printout. Termination for the former cause results in all normal output being printed.

Then the time increment assured is too small the computer round off error dominates the actual calculations and the output is meaningless.

This thin leaves us with the estegory of the time independ to the line independ on the line independent on the effect of the time independent in



clude ballast removal rate, vehicular volocities and accelerations, and vehiclular stability both static and dynamic.

The ballast release rate in pounds per second (real time) can be assumed to be fixed for a given vehicle since, in energency conditions, all ballast will, in general, be released at as high a rate as possible. Even if this is not done the ballast release rate would normally be specified and not left to the needs of the simulation. Further, the program in no way affects the ballast release rate.

The velocities and accelerations, on the other hand are directly affected by the time increment used. The change in velocity is determined by assuming that the acceleration is constant over the time period and is, therefore, equal to the product of the time increment and the acceleration. The acceleration existing during a time increment is dependent upon the forces acting on the body during that increment. The hydrodynamic forces are, in turn, dependent upon the velocities computed during the previous increment. This interdependence of forces, accelerations and velocities can be the cause of premature termination if the time improment is too large. For instance, a large increment would cause the acceleration to act for too long causing the velocity to be excessive, which in turn would cause the forces in the next increment to be excessive, etc....

The static stability of the vehicle is dependent only upon the shift of the CG relative to the CB, and is, therfore, a function of the ballast release rate only. The dynamic stability is, however, dependent upon the vehicular velocities and accelerations as well as the change in mass caused by the reduction in mainht.

The essence of this discussion is that, the time increment to be used for a validate in free ascent will be distincted by a could water of the



rate will indicate the order of magnitud and the accelerations will refine the increment.

With the proper time increments chosen trajectories can be conputed for a variety of initial conditions.

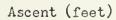
IV - 1.2 DSRV Hotion Touches

The ascent trajectories generated by the ascent program are not those that would be followed by the DSRV, since the coefficients used in the program come from DSRV model tests with the propeller running. They do, however, provide an indication of the ascent trajectories and motion tendancies of that vehicle. Simulations were made with the body axes initially coincident with the inertial axes and with an initial roll angle imposed.

The results of the simulations with the body axes initially coincident with the inertial axes clearly indicate the coupling of pitch and surge which bring about the trajectories of Figure IV.1. This motion becomes even more clear in the velocity plots of Figure IV.2. A tendancy toward a regative pitch angle when dusping both train tanks into the reservoir is also comonstrated.

The simulations involving an initial roll angle indicate the small amount of roll damping and static stability associated with this vehicle. They also indicate a slight side force roll coupling as is evidenced in the printest in Appendix C, section C - 2. The lack of stability also manifested itself by causing the program to terminate prematurely when using a time increment that proved successful for the zero roll case. Eventually an increment of one tenth that used without roll proved successful.





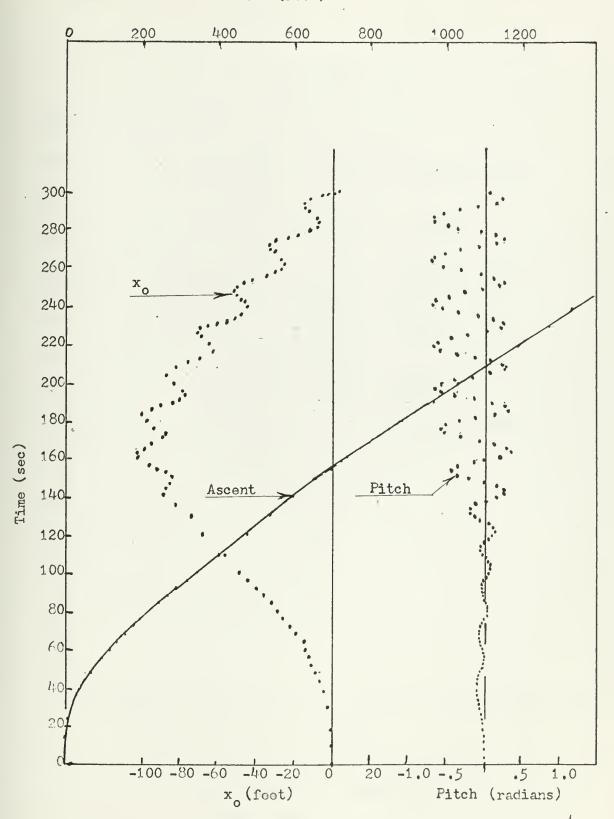


Figure IV.1



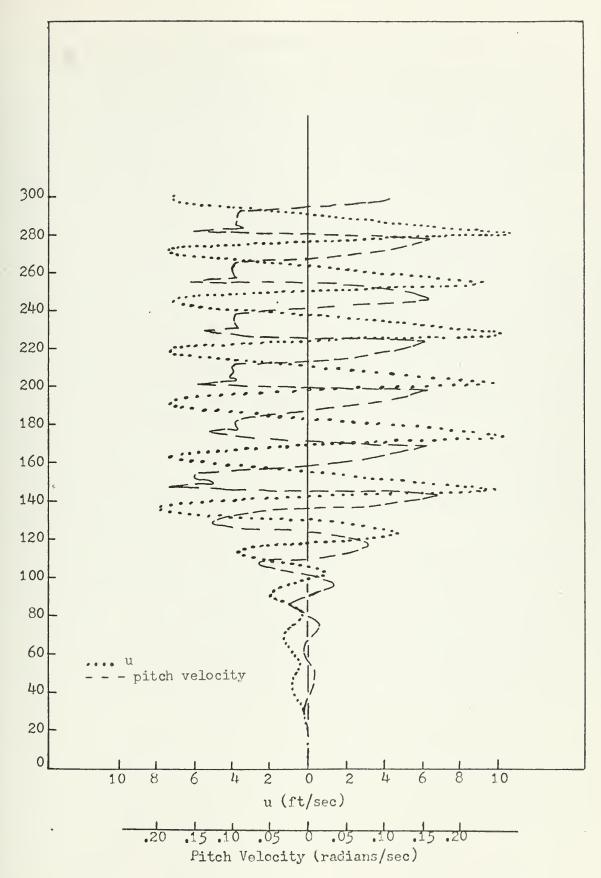


Figure IV.1



IV - 2 Conclusions and Illio Lendations

The computer simulations using the DSRV characteristics have denonstrated the worth of this program in determining the motions of a vehicle as it ascends under the influence of buoyancy alone. They have shown that a vehicle such as the DSRV is quite sensitive to roll without the stabilizing effect of a relatively large forward velocity.

The velocities achieved by the vehicle during ascent indicate that there is a predominant velocity component about which we could expand a Taylor series once the initial acceleration phase of the ascent has been passed. The most likely time for a shift to this sort of formulation is twenty seconds after deballasting has been completed. At that time the vehicle appears to have reached a terminal condition in which the surge and pitch vary between essentially constant limits.

The simulations have indicated that a vehicle with near perfect port and starboard symmetry requires no more than two discussional equations of motion when there are no side forces present.

It is recommended that future studies to made using coefficients obtained from model tests conducted without propellars numning. Also, in view of the slow ascent velocities, it may prove vertibilite to rewrite the equations to include the effects of an ascent propulsor which may prove necessary in order to get a more rapid ascent in other than congency situations.



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APPE IDIX A

COMPUTER PROGRATIFOR ASCENT TRAJECT MILES

A - 1 General:

The computer program to calculate free ascent trajectories is written for the 184 360 - 65 digital computer of the HIT Information Processing Center. The program is written in FORTHAM IV (see ref 11 and 12). The program makes use of the matrix manipulation submoutines contained in version III of the IBM "Scientific Submoutine Package" (ref 13) and the mathematical functions contained in the standard "Library Submoutines" (ref 14).

The system subroutines required are:

Library Submoutines;

SIN - computes the sine of an angle

COS - computes the cosine of an angle

TAN - computes the tangent of an angle

SQUT - computes the square root of a number

ABS - computes the absolute value of a nu ber

Scientific Subroutines;

GHPRD - computes the product of two multices

MINV - computes the inverse of a matrix

Extensive use is made of the NALELIST feature of FURTIVE IV for input and output.

The program is divided into seven parts: the MAIN program and six SUBROUTINES.



A - 1,1 IAIN Fromer

The HAIN program acts as an executive reading in initial conditions, vehicle parameters and program control parameters, calling the various subroutines needed to solve the equations of motion and writing out the results of the computations.

The data read in is contained in five UNVELISTS, MCCom, FLUID, VIRITS, VIRICL and COMMAL, and one controlled format statument. The entput is again in accordance with the NALEIISTS plus a single large array, POSIT, containing all the computed results for the entire test paried.

Part of the output from MANNIELTST ILCOLD. This output can be used as input data for another run in order to continue the same trajectory if it is desired. A regimen of five hundred steps is allowed in one run.

A - 1.2 Subroutine CUMF

COMPT reads in the non-dimensional coefficients and immediately prints them for output. The subroutines then computes dimensionalizate factors, dimensionalizes the coefficients for use in the motion equations and prints the dimensionalized coefficients for output.

The coefficients are read in and printed using the NOECIST COTFFS.

The dimensional form.

A - 1.3 Sub outing PAIAST

BALAST computes the changes in weight, mass, center of gravity, and moments of inertia due to the dropping of ballast.

This subroutine is only called during the initial phase of the



trajectory when deballasting takes place.

BALAST calls submoutine AMATRX since the entrice generated in AMATRX varies only during the deballasting phase of the ascent.

A maximum of twenty locations for ballact weights is allowed.

There is no printed output from BALAST.

A - 1.4 Submouting AMATEX

AMATRA computes the six by six matrix of acceleration term coefeficients, AA, and then inverts the array for use in the solution of the
equations of motion.

A - 1.5 Subrouting HYDRC

HYDRO computes the hydrodynamic forcing terms and the gravity forcing terms and then sums them, plus any effector forces, to get the total forcing terms for the equations of motion.

HYDRO uses the velocities and angles resulting from the previous steps in order to compute the hydrodynamic and gravity force components acting on the vehicle during the present step.

There is no printed output from HYDRO.

A - 1.6 Subroutine TRAJEC

TRAJEC computes the translational and rotational velocities of the vehicle in both body fixed and earth fixed axis systems. In order to do this, TRAJEC also computes the translational and angular velocity transfermation matrices AINV and THNV.

If the pitch angle becomes ninety degrees the transformation nation, Alev, will blow up (nathematically speaking). At this time, the program will print a message saying that this has occurred. At the same time, the computer system will issue a "FBS intercept, Divide check" reasons



during computation of the two elements in ALT which involve division by the cosine of the pitch angle. At this point the system will take the standard corrective action of assigning the value of 10° 59 to each of these elements.

A - 1.7 Subroutines POSITY

POSLITM computes the time ellapsed and the position of the volicle relative to the earth fixed origin. The vehicle position includes both translation and rotation. POSLITM also stores all the trajectory generation information in the array POSLITM for the final output sequence.

There is no printed output from POSTIM.

A - 2 Inout Culout

The variables and constants needed as input for this co pater program are read into the program by seven read statements. Table 1 lists all the input variables and the formats under which they are read.

A single computer run must be rade to simulate each validate. If, however, a run terminates before the vehicle resches the surface, the punched output from the program gives the imput necessary to continue the trajectory on the next run.

The program output gives a time history of the simulation. This data is stored in an array during execution and printed at the end of the run by the main program. During program execution NASSIST/INCCLD/ is printed at the completion of each step. This is done to aid the user in locating the source of any premature program termination. Premature termination is generally caused by incorrect input data.

The dimensions of the storage erray restricts the program to 500 time intervals, however, the program storage require rate one such that this dimension can safely be increased to 1000. This, coupled with the



ability to continue a trajectory from our run to the not, allow an un-

For sample output see Appendix C.



TABLE 1

IFPUT TO ASCENT PROJUCE

Columns	Format	Symbol	Description
2 - 8		\$INCOID	This namelist will take more than one card and
			includes the variables described in Table 2.
			The list begins with \$IECOAD and ends with
			\$END.
2 - 7		\$FLUID	This namelist will take one card and includes
			the variables described in Table 2.
2 - 8		\$VIMITS	This namedist will take one card and includes
			the variables described in Table 2.
2 - 8		\$VEHICL	This nerelist will take one card and includes
			the variables described in Table 2.
2 - 8		\$CONTRL	This namelist will take one cord and includes
			the variables described in Table 2.
2 - 8		\$00 <u>1</u> 245	This namelist will take more than one cond and
			includes the variables listed in Tables 2 and
			3.
1 - 10	F 10.0	XI	The position of the bellast weight relative to
11 - 20	F 10.0		the origin of the body sais coordinates.
21 - 30	F 1.0.0	ZZ	(feet),



Colums	Format	Symbol.	Description
31 - 40	F 10.0	D. W22	The rate at which the particular ballest
			weight is removed (pounds per second).
41 - 45	I 5	IXXI	The number of time steps during which a
			particular hallast weight is removed. There
			is one card of this type for each position
			from which ballast is removed.



A - 3 Description of the liets

The following tables contain the MA mission und by the pregram.

TABLE 2

MARLISTS

Mamo List/IKCOLD/

PROGRAM VARTABLE		DEFINITION & UNITS
ISTEP		The step number
DT	Α¢	The time interval used during deballesing (seconds)
TIME	t	The time elapsed since ascent cormonced (seconds)
XE	x _E	The x component of the distance traveled relative to the inertial origin (feet)
YE	\mathcal{Y}_{E}	The y component of the distance traveled relative to the inertial origin (feet)
Zes	Z _Ľ	The z component of the distance traveled relative to the inertial origin (feet)
РП	¢	The angle of roll (radians)
THETA	θ	The angle of pitch (radius)
PSI	Ý	The angle of you (radians)



FIGURANT VARIABLE		DOMETERON & U. 115
VEARTH	0	The vehicle vector of translational valority in
		inertial coordinates (feet per second)
VEARTH	6	The vehicle vector of angular velocity in inertial
		coordinates (radians por second)
BVEL	V	The vehicle vector of translational velocity in
		the Mody axis system (feet per second)
BROT	W	The vehicle vector of angular velocity in the Body
		axis system (radians per second)
WT	W	The vehicle weight (pounds)
XG	$x_{\overline{G}}$	The x compenents of the CG vector (feet)
ΥG	\mathbf{y}_{G}	The y component of the CG vector (feet)
ZG	z _G	The z component of the CG vector (feet)
В	В	The vehicle buoyant force (pounds)
XB	x_{B}	The x component of the CB vector (feet)
Y B	y_{B}	The y component of the Cr vector (feet)
2.8	Z ₃ 3	The z component of the CD vector (feet)
XXI	II,	The ross notion of interior (boot the wests (fings)
TYY	J. A. D.	The mass in what of inertia cloud the year is (sheet)
XZZ	I 22	The mass now reach in rise there is note (Flogs)



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Name List/CC 100/

PROGRAT	USUAT ROJERCIATURA	DENTRITION & ULIUS
XEFF	$X_{\underline{\mathcal{B}},\mathcal{F}_{\overline{\mathcal{F}}}}$	The X component of the effected ferros (por is)
XEFF	YEFF	The Y component of the effector forces (pounds)
ZEFF	$z_{ m EFF}$	The Z component of the effector forces (pounds)
KEFF	KEFF	The K component of the effector forcer (periods)
FREE	HEFF.	The A component of the effector forces (pounds)
NEFF	N _{EFF}	The N component of the effector forces (punnds)
		Name Mist/COEFFS/
X	X i.j	Longitudinal forces eccificants for a vehicle
		moving ahead
XA	XA	Longitudinal force confficients for a vehicle
		noving astern
X	Y	Lateral force coefficients for a valuals moving
		ehoad
YA	УA ij	Lateral force coefficients for a volticle nowing
	v	astern
Z	Z ij	Normal force coefficients for a vehicle making
		ahoad
ZA	7.^ i.j.	For 1-1 force coeffici wis for a valid noting
		astern



THE RY I	USUAL NO. IDECLATURS	Dath 1. 10. 5 C U 110
K ·	Kaj	Roll nonemt force conflictable for a vehicle new-
M	KA _{ij}	Roll Horast force confrictants for a velicle nov-
ri	H.j., j	Pitch No. ent forse coefficients for a vehicle movin; als d
114	1:A <u>.</u> ;	Pitch Lowest force coefficients for a vehicle moving estern
Īζ	^N ij	Yam Howert force coefficients for a validle nov-
HA	NA ij	Yaw Homent force confined this for a vehicle roveing astern

The subscripts i, j refer to the possible combinations of the body velocities and accolerations as described in Table 3.



TABLE 3

PROJRA. 1	STAND	APO (PSTEC)	SUDSCITET IN			
SUBSCRIPT	X, XA	Y, YA	Z, ZA	K, KA	Tr, Tu	N, IIA
1.	นน	VV	u or *	V V	u	y v
2	VV	V	VV	V	VV	
3	1307		$F_{k} \big[(\mathbb{S} \big[$	∇w	$M[V_i]$	VI
L _F		Уtī	\mathcal{D}_{λ}	uu or *	V	V
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13						lvlx
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UEAL K, V. N. KEFE, EFE, NEFE, TXX. TYV. IZZ, TYZ. ZA. GA, NA JI H. STO. DESTT (51,500), ANT (7.5), X (15), Y (1.5), Z (1.8), X (11), N (1.5), Y (1.5), Z (1.8), X (11), N (1.8), X (1.

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PEADICS.SCO.) (XI(I),YI(I),ZI(I),OHASS(I),IMAXI(I).I=1,IMAX)
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(CALL POSITIN

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3N - L.J. T. D. X. SETT , V. GETE, Z. GETE, G. PEFF, V. PEFF, X. S. Y. S. S. S. V. F. F. T. G. A. S. S. A. T. A. J. S. V. S. V. S. V. F. Z. F. D. A. S. S. A. T. A. V. A. A. V. A. V
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IF (I.A. .L. ...) Q.TUP?
IF (ISTED-1 .3T. (AAK((I.AY))) TAK
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2XA(1 ),V((2)),2A(13),K1(13),7((11),7(11),7(2),Y1(2)),Y1(20),Z1(20),
3EAL 4,5. A, KTEL, (EPT, NEFF, IXX, TYY, 122, IXZ, K, (3, N) 01, EAS SIN LI(C), LM(A)
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IVW = A3SISCRT (VV+"2)

 $\begin{array}{l} M4Y) = -1X2 \times \{ \ Rx - PP \ \} + \{ \ 122 - 1XX \ \} + W(1) \times JJ \\ 2 \ V^{2} (S \times \{ \ YG \times \{ \ US - VP \ \} + ZG \times \{ \ US - YP \ \} + W(1) \times JJ \\ 3 \ Y(2) \times VV + W(3) \times JVV + W(4) \times JJ + W(5) \times JJ + W(5) \times JJ + W(1) \times JJ \\ 4 \ Y(5) \times JVM + W(9) \times VP + W(10) \times VX + W(11) \times JJ + W(12) \times JJ \\ 5 \ O + V(17) \times JP + W(16) \times JV + W(17) \times JV + W(11) \times JP \\ \end{array}$

5 XHVD = VV5SS × (WV = 12 + X3 × (12 + 02) + V6 × P9 + V5 × RP) + XA(11) × VV + XA(2) × W + XA(15) × W + XA(15) × W + XA(15) × W + XA(16) × W + XA(16) × W + XA(16) × W + XA(17) × W + XA(17) × W + XA(18) × W + YA(18) × W +

C: 2 (CI) 12+ 5

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00TP(1) = VIY0 + X6x4V + V0TP(2) = VPV) + V0x1V + V0x1
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15 S = SIN(T45T4)

CT = CCS(T48T4)

CS = CT * S1((2H1))

CC = CT * COS(2H1)

**R = AT = 3
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PCAL K,U,N,KFFF,WFFF,NEFF,IXX,TYY,1ZZ,1ZZ,4ZA,KA,MA
DIMINSION AINV(3,3),TIMV(3,2)
ETVENSION PCSIT(31,532),AH(5,6),K(18),Y(16),Z(16),K(18),A(18),N(18
2),99T(3),BPT-(6),VFATH(2),ACAKTA(3),PVFL(3),APCT(4),F(13),C(14),
2XA(12),YX(13),ZA(13),KA(10),AA(18),AA(18),XI(20),XI(20),ZI(22),
K.D.N.KEFF.WFFF.NEFF.IXX.IVY,IZZ,IXZ,KA, AA.NA
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AP. DLY B

TEST VEHICLE GEOLETICS AND COLUMN CHARASTICS

The stability and control coefficients used in the computer sixulation of a free ascent trajectory cone from a report on the stability and
control characteristics of the Deep Subscripts of Recone Vehicle (see ref 15).

These coefficients were obtained using a model with propeller running, which
violates one of the restrictions originally imposed on the problem. This,
however, presents no problem since the coefficients with propeller remains
can be assumed to represent some flotteious vehicle without propeller run
ning. This ficticious vehicle would be similar to the FDEV but would effectively have additional lifting surface on fin aft.

The coefficients and desiratives, Bisted in Tables 4, 5, and 6, were experimentally and analytically obtained by 1900. There exciticient will be supplimented by the terms which can be estimated for the potential theory of Chapter II (see Table 7).

The drag coefficient, to be used in conjunction with the car disconsistent ascent trajectory computation, is obtained from a plot of normal force coefficient as a function of angle of attach found in reference (15).

This coefficient is taken as 0.005 at an angle of attach of 90 degrees.

The geometric characteristics, listed in Table 8, are again those for the DSRV and come from reference (15).

The ballest release rates cone from reference (5) and are listed in Table 9. The only weights that can be released from the DELV are the moreony from the trim and roll systems and the water contained in the vaniable ballest tanks. The moreony must all pass through the convent reservoir tank to be dropped, therefore, the moreony release with its first in the



release rate from the resorroir. The variable bullet test and asset to be full at the beginning of the ameent sine; this is the our send up a distance the our send up a distance this is the o



Vertical - Plane Stability and Contabl Larivations

	Michig	And
71 *	~0.011310	0.011973
Q ·	-0.017455	0.015420
J	-0.000146	-0.000107
K.	-0.021545	0,029515
Z.º	0.001573	-0,001321
Q Z	-0.000i30	-0.000250

TABLE 5

Horizontal - Plane Stability and Control Land winves

14 °	· •0 • 01.2497	-0.014732
Y ·	0.025955	- 0.016795
K.	-0.00020	0.000380
r.	0.000182	.0.000109
·	-0.035545	=0.010011
Y V	0,000190	0.000175
N	•0.001531	-0.001352
Y ·	U,000100	·0.000250
V a V	-0,000012	0.000063
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Note: resitive direction of body and considered the second or ball sheet and extern notion.



Stability and Combrol Coufficients from Curve

Fitting and from Jetherica

H _{sc}	0.000081	K_{π_0}	-0.00157
14	0.011175	K^{Λ}_{ϵ}	0.002216
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J.1	-0.003419	K P	.0.000157 ¹¹
M .	~0. 003624	K.:	-0.000092-
V[W]	-0.023017	K P P	-0.000037
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Z	~0.0 43938	X *	-0.007!!!2
Z .	0.003937	X _{VV}	0.017510
Z *	~0 ,020773	X	O 2 Oxigo
Z *	-0.030243	X. e	-0.067306
	40,082739	YVIVI	-0.173503
n. V	-0.02(053		0.036008



Coefficients Esti ted on the Peris of Potential Temp

	$\Delta h \simeq 2$	1 (7)
$\chi^{d\dot{a}}$	-0,000130	~0:000250
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У	0.000130	0,000250
rq Z _{vp}	-0.035545	-0.0389'15
vp Z	0.000400	~0.000380
z rp K	0.004.000	0,009400
K VVI V.	0.000270	-0.000530
VQ K	~0. 000270	0.00350
J. T.	0.00001/2	~0.00000
Eq.	0.00001/2	-0.000131
K Qr	-0.000°°00	0,000730
T) VP	0.000012	-0. 00006
i Tr	0.000042	~0.00008
K rr	0.001432	0.001260
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A. L. J. J.

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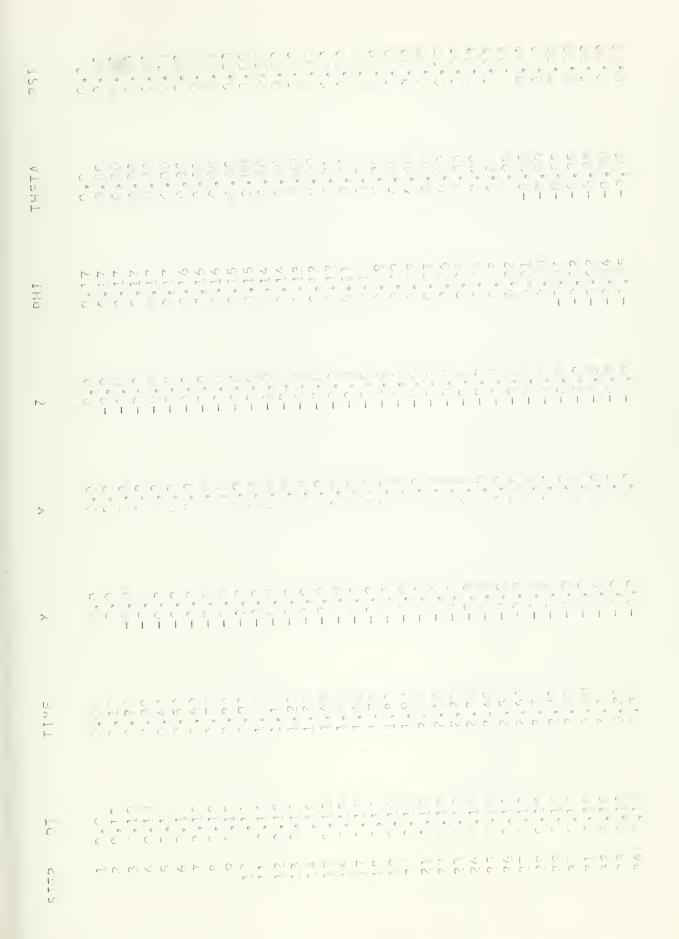
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A TREAD TO BE TOUR COLD AND AND A TRANSPORT

In order to occupate the or distributed ascent to jectory developed in Chapter III, the program listed on the following page was written.

This programments in initial conditions and deballasting parameters, computer the acceleration valuably and distance travelship the variable.



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ONE DEVENSIONAL ASCENT TRAJECTORY GENERALDS PARTITIONAL IST/INCO/WF/B, Disk, Z, DR, A, D, OT, W. N.
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STEP	WEIGHT	4 3UMY 1	1001	1.1	1 - 1 - 1 - 1
1	140000.00	35,23	-0.04	-0.00	
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3	140000000	105.70	-0.013	() * () s	-1.71
4	147207.00	147.03	-0.017	-0.022	0, 12
5	140000.00	176.16	-0.022	-0.003	-5.03
6	140000.00	211.40	-0.026	-0.046	-7.75
7	140000.00	246.63	-0.030	-0.061	-0.08
8	140000.00	281.86	-0.935	-0.078	-^.11
9	140000.00	317.10	-0.039	-0.098	1 - 1 5
10	140000.00	352.33	-0.043	-0.119	O e 2 I
1.1	140000.00	387.56	-0.147	-0.143	-0.27
12	140000.00	422.80	-0.05/	-0.169	
1.3	140000.00	453.03	-0,756	-0.197	-11,44
14	140000.00	493.26	-0.060	-0,227	-7.55
15	140000.00	528.49	-0.064	0,258	-0.67
16	140000.00	563.73	-0.063	-0.292	-0.81
1.7	140000.00	598.96	-0.072	-0,328	-0.96
1.8	140000.00	634.19	-0.076	-0.,366	-1.14
19	140000.00	669.43	-0.079	-0.406	-1.32
5.0	140000.00	704.66	-0.083	-0.447	-1.54
21	140000.00	739.89	-0.096	-0.490	-1.78
2.2	140000.00	7.75.12	-0.090	-0.535	-2.03
23	1,40,000.00	810.36	-0.003	-0.582	-2.31
24	1.40000.00	845,59	-0,096	-0.629	-2.32
2.5	140000.00	880.82	mf. 609	-0.679	-2 , 94
2.6	140000.00	914.06	-0.1.72	-0.730	-3.31
27	140000,00	951,29	- Co 11.4	-0.782	-3.57
2.8	140000.00	986.52	-0.106	-0.825	-4.08
2.9	140000.00	1021.75	-0.109	-0.889	-4.51
30	140000.00	1056.99	-0.111	-0.944	-4.97
3.1	140000.00	1092.22	-0.112	-1.001	-5.45
3.2	140000.00	1127.45	-0.114	-1.058	-5.97
33	140000.00	1162.69	-0.11.5	-1.115	-5.51
34	140000.00	1197.92	-0.117	-101/4	-7.08
35	140000.00	1233.15	-(,118	-1.23?	-7.68
3.5	140000.00	1268.39	-0.113	-1.292	-9.32
3.7	1400000.00	1303.62	-0.119	-1.351	-8.98
311	140(1.0.00	1338.85	-0.119	-1.411	-9.67
39	140000.00	1374,08	-0.119	-1.470	-13.39
40	1400000000	1409.32	-0.119	-1.530	-11.14
41	140000000	1444.55	-0.119	-1.500	-11.72
4.2	140000,00	1479.78	-0.119	-1.640	-12.73
43	140000.00	1515.02	-0.118	-1,708	-13.57
4.18	140000.00	1556.25	-0.118	-1.767	-14.43
45	140000.00	1585.43	-0.117	-1.025	-15.33
46	140000.00	1.620 - 11	-0.116	1, 923	-16.26
67	140000000	1655,95	-0.315	-1.941	-17.22
4.8	140000.00	1691.18	-0.114	-1.008	-13.20
49	140000.00	1726.41	-0.112	-2,011	-17.21
50	140000000	1761,65	- 6 1 1	-2,170	-20,25



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STEP	140000.00	1796.88	-0.109	-2.114	-2].
5]	140000.00	1832.11	-0.108	-2.21	-22.62
52	140000.00	1867.34	-0.106	-2.27]	-23.54
53	1400.0.00	1902.58	-0.10%	-2.3?3	-26.69
54	140000.00	1937.81	-0.103	-2.374	-25.36
55		1973.04	-0.101	-2.425	-27.06
5.6	14000.00	2008.28	-0.009	-2.476	-28.29
5.7	146000.00	2043.51	-0.07	-2.523	-79.54
5.8	140000.00	2078.74	-0.095	-2.571	-30.81
59	140000.00	2113.98	-0.094	-2.617	-32.11
60	140000.00	2140.21	-0.092	-2.663	-33.43
61	140000.00	21.84.44	-0.090	-2.798	-34.77
62	140000.00	2219.67	-0.038	-2.752	-36.13
63	140000000	2256.91	0.86	-2.706	-37.52
6,4	140000.00	2200 14	-0.085	-2.838	-38.93
65	140000.00	2325.37	-0.083	-2.879	-40.36
66	140000,00	2360.61	-0.037	-2.920	-41.81
6.7	14,000.00	2395.84	-0.080	-2.035	-43.28
6.8	140000.00	2431.07	-0.078	-2.999	-1,4.77
69	140000.00	2466.30	-0.077	-3.137	-46.28
7.0	140000.00	2501.54	-0.075	-3.075	-47.81
73	140000.00	2535.77	-0.074	-3.112	-49.35
72	140000.00	2572.00	-0.072	-2.148	-50.92
73	140000.00	2607.24	-0.071	-3.183	-52.50
74	140000000	2642,47	-0,069	-3.218	-54.10
75	140000.00	2677.70	-0.068	-3.252	-55.72.
76	140000.00	2717.93	-0.CA7	-3:285	-57.35
77	140000,30	2748.17	-0.065	-3.318	-50.00
78	1,40000.00	2783.49	-0.064	-3,350	-61.57
79	140000.00	2818,63	-0.663	-3.392	-62.35
8.0	140000.00	2853.27	-0.062	-3.412	-64.05
81	140000.00	2839.10	- n. C61	-3.044	-65.76
82	140000.00	2924.33	-1.160	-3.474	-67.49
83	140000.00	2959,55	-1.059	-3.523	-4,7.24
84	140000.00	2994.80	-0.058	-3,533	-71.00
8.5 8.6	140000.00	3030.03	-0.058	-3.561	-72.77
8.7	140000.00	3065.26	-0.057	-3.590	-74.56
83	140000.00	3100,50	-0.055	- 3.618	-75.56
8.9	140000.00	3100.50	-0.053	-3.64%	-73.18
90	140000.00	3100.50	-0.045	-3.661	-30.00
91	140000.00	3100.50	-0.147	-3.6641	-51.34
92	140000.00	3100,50	-0.038	-3.70(-83,60
93	140000.00	3100.50	-0.034	-3.123	-85.55
94	140000.00	3100.50	-0.031	-3.730	-87.41
95	140000,10	3100.50	-1.028	-3.702	-81.26
96	140,000,00	3100.50	-(, (,), +	-2.7.5	-01.16
97	140000,00	31,00.50	-0.023	-3.716	-97.95
98	140 200.30	3100.51	-0.121	-7.07117	06.04
90	140000,00	310, 40	=0.010	-3,750	-96.94
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101	1400000000	310.020	0,015	-3 617	-160.64

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102	140000000	3100.50	-0.014	-3.825	-104.46
103	140000,00	3100.50	-0.012	-3,831	-115.37
104	140003.60	3100.50	-0.013	-3.836	-108,79
105	140001.00	3100.50		-3.840	-117.21
106	140000.00	3100.50	-0.009	-3.844	-112.13
107	146000.50	2100.50	-0.008	-3.848	-114.05
1.68	140000.00	3107.50	-C. (177	-3.851	-115.98
109	140001.00	3100.50	-0,007 -0.006	-3.854	-117.90
110	140000.00	3100.50		-3.857	-119.83
111	1,400000.00	3100,50	-0.005	-3.859	-1.21.76
112	140000.00	3160.50	-9.005 -0.004	-3.852	-123.69
113	140000.00	3100.50		-3.854	-125.62
114	140000.00	3100.50	-0.004 -0.004	-3.865	-127.55
115	140000.00	3100.50		-3.357	-129.49
116	140000.00	3100 50	-0.003	-3.868	-131.42
117	140000.00	3100.50	-0.003	-3.870	-133,35
118	140000.00	3100.50	-0.003	-3.871	-135.29
119	140000.00	3100.50	-0.002	-3.872	-137.23
1.20	140000.00	3100.50	-0.00?	-3.873	-139.16
121	140000.00	3100.50	-0.002	-3.874	-141.10
122	1400000.00	3100.50	-0.002	-3,874	-143.04
123	140000.00	31,00.50	-0.002	-3.875	-144.97
124	140000.00	3100.50	-0.001	-3.876	-145.91
125	140000000	3100.50	-0.001	-3.876	-143.85
1.26	14000000	3100.50	0.001	3.877	-150.79
127	140000.00	3100.50	-0.001	-3.877	-152.73
128	149000.00	3100.50	-0.001		-154.66
129	140000.00	3100.50	-0.01	-3.878 -3.878	-154.60
130	140000.00	3100.50	-0.001	-3.878	-159.54
1.31	14000 000	3100.50	0.001	-3.879	-167.48
132	140000.00	3100.50	-0.001	-3.879	-162.42
133	140000.00	3100.50	-0.001	-3.879	- 164.36
134	140000.00	3100.50	-0.000	-3.979	-166.30
135	140000.00	3100.50	-0.000	-3.896	-168.24
135	140000.00	3100.50	-0.000	-3.880	-170.18
137	140000.00	3106.50	-0.000	-3.881	-172.12
138	140000.00	3100.50	-0.000	-3,88C	-174.06
139	140000.00	3100.50	-0.000	-3, 58	-175.60
140	140001.00	3100.50	-0.000	-3, 431	-177.94
141	140000.00	3100.50	-0.000	- 3.330	-170.83
142	140000000	3100.50	-0.000	-3,891	-181.82
143	140000.00	3100.50	-0.000	-3.901	-133.76
744	140000.00	3100.50	-0.000	-3.341	-185.70
145	140,700,00	3100.50	-6.000	-3.633	-127.54
145	140000.00	3100,50	-(-())	-2.501	-139.58
147	140000.00	3177.50	-(.000	-3,531	-171.52
148	140000.00	3100.50	-0.000	3, 8(1	-193.46
149	14000 .00	3101.51	-0.000	-3.811	-195.40
150	140000.00		- ((()	-3 881	-197.31
151	1,0000000	3107,50	-0.000	-3.81.1	-197.28
152	1400000000	3100.50	-0.00-	7 () , 1	



STED	MEI (HI	+ 3UPY 11.Y	, mat	1	7 = 1 1 2 2
153	140000000	310 50	-0.000	-3,1723	-201.22
154	100000.00	3100.50	-4.600	3 . 8 11 1	273.17
155	14,0000,00	3100.50	-0.0011	-3.861	-215.11
156	140000,00	310 .50	-0.000	-3.881	-207.05
157	140000.00	3100.50		-3.881	-208,99
158	140000.00	3100.50	-0.000	-3,881	-210.93
159	140000.00	3100,50	-0.000	-3,881	-212.87
160	140000.00	3100,50	-0.001	-3.881	-214.81
161	1400000000	3100.50	-0.000	-3.881	-216.75
162	140000.00	3100.50	-6,000	-3.981	-218.69
163	140000.00	3100.50	-0.000	-3.881	-221.73
1.64	140000.00	3107,50	-0.000	-3.831	-277.57
165	140000.00	3100.50	-0.000	-3.891	-224.51
166	140000,00	31,00.50	-0.000	-3.881	-226.45
167	140000.00	3100.50	-0.000	-3.881	-226,39
168	140000.00	3100.50	-0.000	-3.881	-230.33
169	140000.00	3100.50	-0.000	-3.881	-232.28
170	140000.60	3100.50	-0.000	-3.881	-234,22
171	140000.00	3100.50	-0.000	-3.891	-236.16
172	140000.00	3100.50	-0.000	-3.881	-238.10
173	140000.00	3100.50	-0.000	-3.881	-240.04
174	140000.00	3100.50	-0.000	-3.881	-241.98
175	140000.00	31:00.50	-0.000	-3.881	-243.92
1.76	140000.00	3100.50	-3,000	-3.881	-245.86
177	140001.00	3100.50	-0.000	-3.881	-247.86
178	140000.00	3100.50	-0.000	-3.831	-240.74
179	140000.00	3160.50	-0,000	-3.501	-251.68
180	140000.00	3100.50	-0.000	-3.881	-253.62
181	140000,00	3100.50	-0.000	-3.881	-255.50
182	140300.00	3100.50	-0.000	-3.881	-257.50
183	140000.00	3100.50	-0.000	-3.881	-259.44
1.84	140000.00	3100.50	-0.000	-3.881	-261.29
1.85	140000.00	3100.50	-0.000	-3.881	-263.33
186	140000,00	3100.50	-0.000	-3.881	-265.17
187	140000.00	3100.50	-0.000	-3,881	-267.21
188	140000.00	3100.50	-0.000	-3.881	-259.15
189	140000.00	3100.50	-0.000	-3.881	-2.71.09
190	140000.00	3100.50	-0.000	-3.881	-273.03
191	140000.00	3100.50	-0.000	-3600	-274.97
192	140000.00	3100.50	~0.000	-3.881	-276.91
193	140000.00	3100.50	-0.000	-3.801	-273.85
194	140000.00	3106.50	-0.000	-3.901	-237.10
195	140000.00	3100.50	-0.000	-3.801	-217.73
196	140000.00	3100.50	-0.000	-3.891	-234.57
199	140000.00	3100.50	-0.000	-3.881	-234.61
198	140000.00	31(.0,50	-0.000	-3,981	-289.55
100	140000.00	3100.50	-0.000	-3.887	-20 .50
200	140000.00	3100.50	-0.000	-3.501	-207014
201	140000.00	3100.50	-0.000	-3.931	-204.38
202	140000.00	3,00,50	-0.000	-3.981	-276.32
203	140000.00	3100,50	-0.0.	-3.993	-211.26
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